Small-x Helicity Evolution: New Global Analysis with Polarized DIS and SIDIS Data

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SURGE COLLABORATION

Proton Spin Puzzle and Small-x

- Proton Spin ≡ ½, Measured Quark Spin + Gluon Spin ≠ ½
- Jaffe-Manohar sum rule [1]: $S_q + L_q + S_G + L_G = \frac{1}{2}$

$$S_q(Q^2) = \frac{1}{2}\int_0^1 dx\, \boldsymbol{\Delta\Sigma}(x, Q^2), \quad S_G(Q^2) = \int_0^1 dx\, \boldsymbol{\Delta G}(x, Q^2)$$

- $x \equiv$ Longitudinal momentum fraction.
- Experimental data only exists for a range of values $x > x_{min} > 0$.

- Helicity Parton Distribution Functions (**hPDFs**)
  - **Helicity ↔ Spin**
  - $\Delta G \equiv$ Spin contribution from gluons as a function of x
  - $\Delta\Sigma \equiv$ Spin contributions from quarks as a function of x

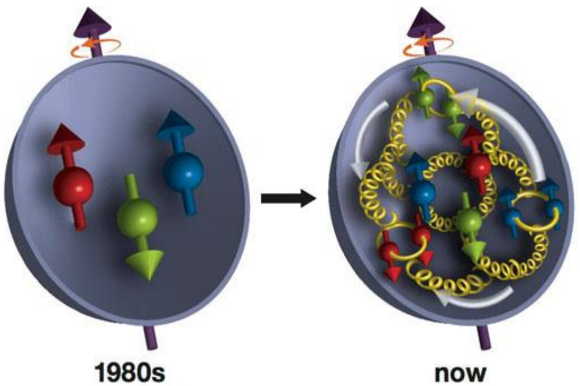


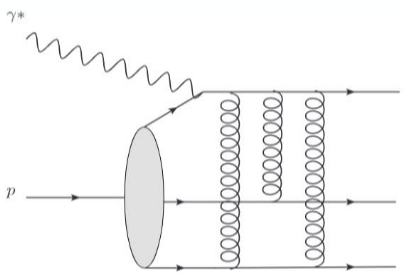
Image from Brookhaven National Labs

Small-x Helicity ⇒ Describe $\Delta\Sigma$ and ΔG as $x \to 0$

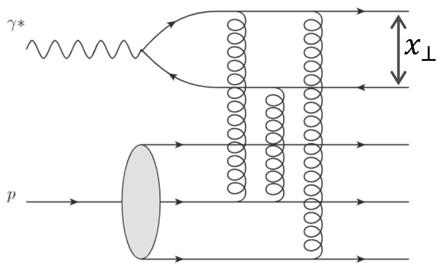
Dipoles and Double-Logarithmic Approximation

Deep Inelastic Scattering: $l + p \to l + X$

Large-x: "Knockout"



Small-x: "Dipole"

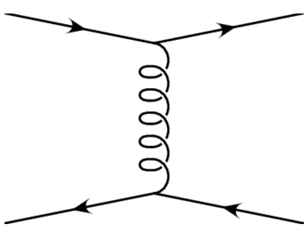


At small-x, high momentum quarks and gluons are very time-dilated and scatter coherently.

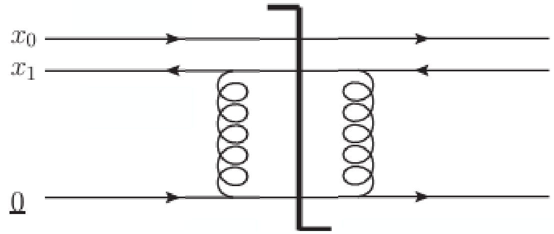
$$x \propto \frac{1}{E^2} \Rightarrow x \neq 0 \text{ from Experiment}$$

Two small-x partons make a **dipole** [3].

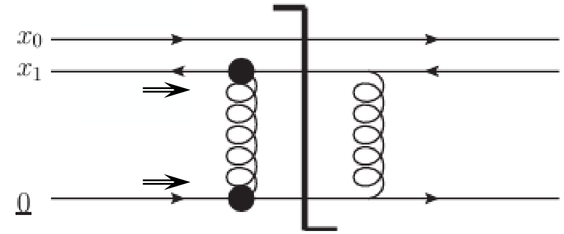
Simple Scattering Amplitude



Unpolarized Dipole Amplitude



Polarized Dipole Amplitude



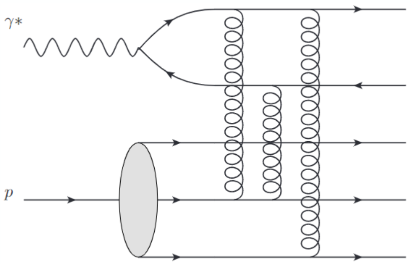
KPS-CTT DLA Evolution Equations

KPS-CTT = Kovchegov-Pitonyak-Sievert-Cougoulic-Tarasov-Tawabutr [2], [6]

- Evolution is only solvable if we assume N_c or $N_c \& N_f$ are large.
  - $N_c \equiv$ # of Colors, and $N_f \equiv$ # of Flavors.
  - Before SURGE: large-N_c, pure gluon limit
  - With SURGE: large-$N_c \& N_f$, quarks restored, more realistic
- Running Coupling: α_s scales with transverse dipole size.
- New Flavor Non-Singlet Evolution: Used to fit to Semi-Inclusive DIS (SIDIS) data [4].

$$\boldsymbol{\alpha_s \ln^2 \frac{1}{x} \approx 1}$$

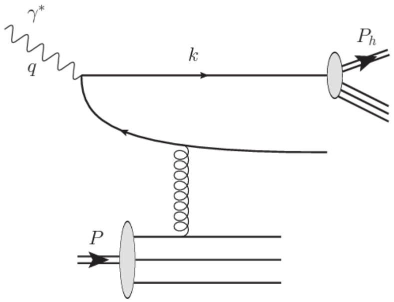
Small-x DIS



Sensitive only to the combination:

$$\Delta\Sigma = \Delta q_f + \Delta\bar{q}_f$$

Small-x SIDIS



Sensitive to specific quark flavors:

$$\Delta q_f$$

Helicity Distribution and Polarized Dipoles

- There are 5 **flavor singlet** polarized dipole amplitudes: $\boldsymbol{Q_f}, \boldsymbol{\widetilde{G}}, \boldsymbol{G_2}$, and 3 **flavor non-singlet** polarized dipole amplitudes: $\boldsymbol{G_f^{NS}}$, where $f = u, d, s$.

$$\boldsymbol{Q_f}, \boldsymbol{\widetilde{G}}, \boldsymbol{G_2}, \boldsymbol{G_f^{NS}}(x = 0.1) \;\; \Rightarrow \;\; \textbf{KPS-CTT Evolution} \;\; \Rightarrow \;\; \boldsymbol{Q_f}, \boldsymbol{\widetilde{G}}, \boldsymbol{G_2}, \boldsymbol{G_f^{NS}}(x < 0.1)$$

- The **flavor-singlet** hPDFs can be written in terms of these polarized dipole amplitudes:

$$\boldsymbol{\Delta G} \propto \boldsymbol{G_2}$$

$$\boldsymbol{\Delta\Sigma} = \Delta q_f + \Delta\bar{q}_f \propto -\int\int\left[\boldsymbol{Q_f} + 2\boldsymbol{G_2}\right]$$

- The **flavor non-singlet** combination of quark hPDFs can also be expressed [4]:

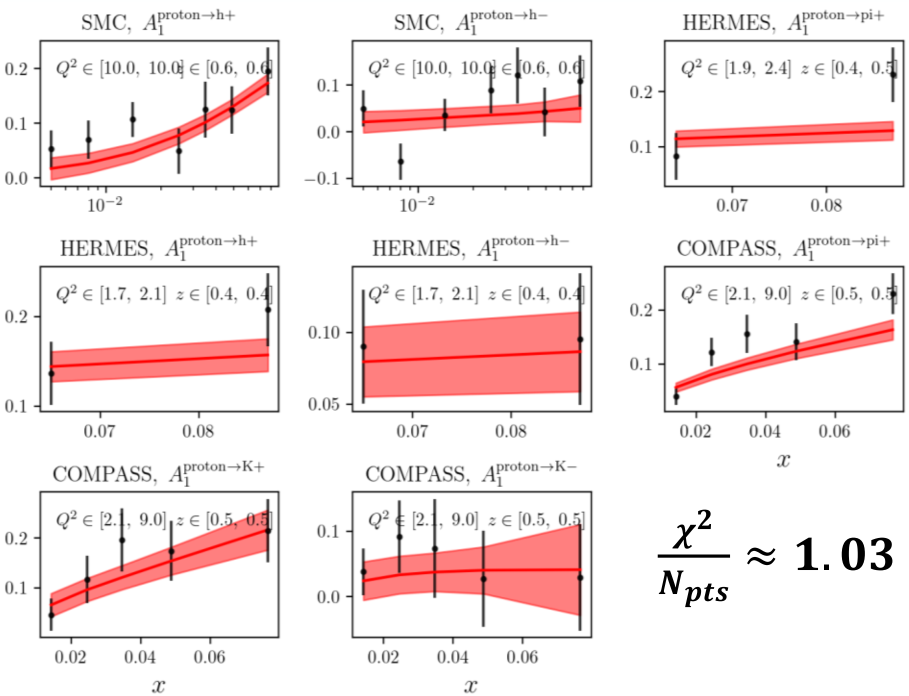
$$\boldsymbol{\Delta q_f^{\mathrm{NS}}} = \Delta q_f - \Delta\bar{q}_f \propto \int\int\left[\boldsymbol{G_f^{NS}}\right]$$

$$\boldsymbol{\Delta\Sigma}, \boldsymbol{\Delta G}, \boldsymbol{\Delta q_f^{NS}}(x = 0.1) \;\; \Rightarrow \;\; \textbf{KPS-CTT Evolution} \;\; \Rightarrow \;\; \boldsymbol{\Delta\Sigma}, \boldsymbol{\Delta G}, \boldsymbol{\Delta q_f^{NS}}(x < 0.1)$$

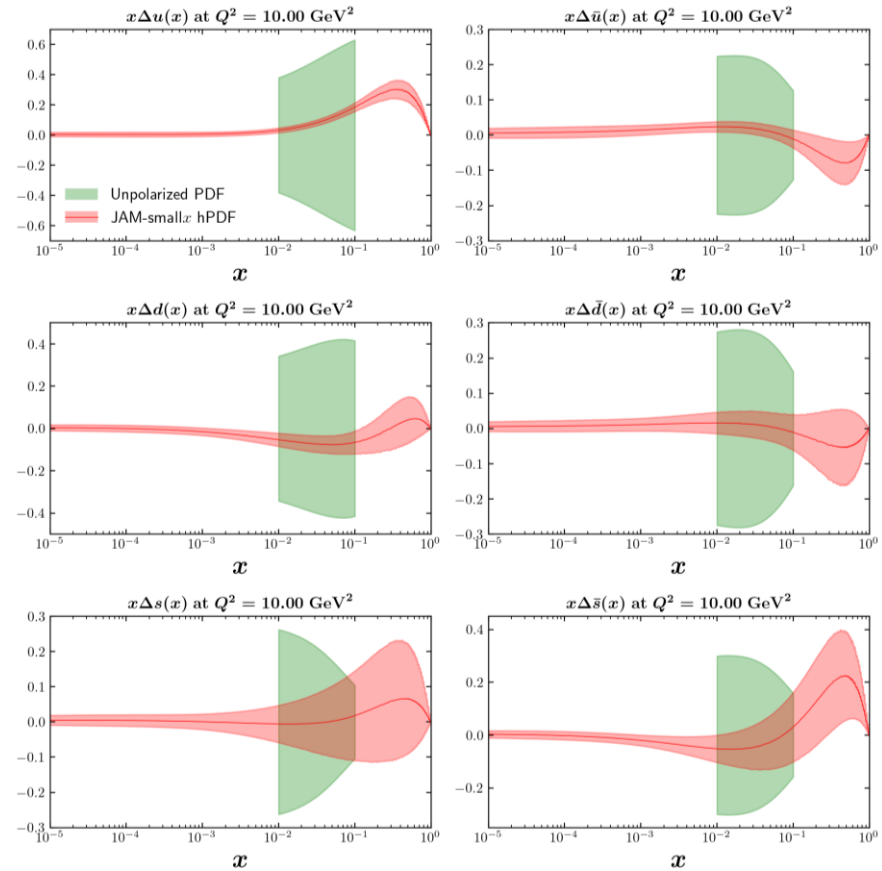
Global Analysis: Data vs Theory

We solve the evolution equations and fit the result to DIS and SIDIS data at moderately small x, N_{pts} =226 data points, with good agreement [7].

Data vs Theory: Subset of SIDIS Data



$$\frac{\chi^2}{N_{pts}} \approx \mathbf{1.03}$$



Incorporating SIDIS data allows us to separate quark flavor hPDFs, $\boldsymbol{\Delta q_f}$, from $\boldsymbol{\Delta\Sigma}$.

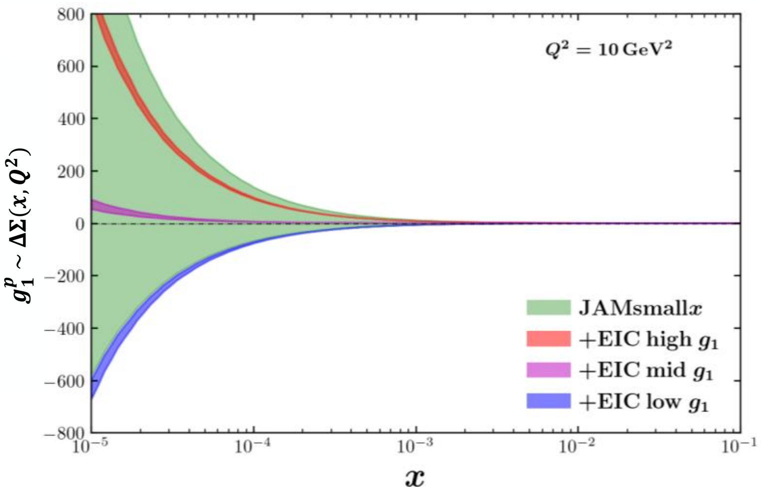
Predictions, Extractions, and the EIC

Total Spin from small-x partons:

$$S_{q+G}\Big|_{x=10^{-5}}^{x=0.1} = \mathbf{-0.64 \pm 0.60}$$

Potentially **large** and **negative** contributions from small-x partons [7].

$$S_q + L_q + S_G + L_G = \frac{1}{2}$$



- DIS and SIDIS data are insensitive to the small-x sign of the hPDFs
- Polarized $p + p$ Collision data is more sensitive to the sign.

Electron Ion Collider Impact: Fits using generated pseudo data consistent with the EIC's yellow report [8].

- Eliminates sign ambiguity and maintains control over uncertainties as x approaches zero.

Review of SURGE Spin WG Milestones

Small-x Helicity Milestones:

- Year 1: Include **running coupling** corrections into **large-N_c&N_f** helicity evolution.
- Year 2-3: Icorporate **SIDIS** and **polarized $p + p$** collision data into small-x helicity analysis.
- Year 3: Make predictions for quark and gluon **Orbital Angular Momentum** at small-x

Published Work [7]:

- Improved small-x helicity evolution with the **Large-N_c&N_f** limit.
  - First Global analysis was large-N_c only with **running coupling**.
- Global analysis of small-x DIS and **SIDIS** data.
  - First global analysis was DIS only.

Active work:

- Incorporating **polarized $p + p$** collision data into DIS+SIDIS global analysis.
- Small-x **orbital angular momentum** phenomenology.

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[1] R. L. Jaffe and A. Manohar,The G(1) Problem: Fact and Fantasy on the Spin of the Proton, Nucl. Phys. B337 (1990) 509–546.

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[4] Y. V. Kovchegov, D. Pitonyak, and M. D. Sievert, Helicity Evolution at Small x: Flavor Singlet and Non-Singlet Observables, Phys. Rev. D 95, 014033 (2017).

[5] D. Adamiak, Y. V. Kovchegov, W. Melnitchouk, D. Pitonyak, N. Sato, and M. D. Sievert, First Analysis of World Polarized DIS Data with Small-x Helicity Evolution, Phys. Rev. D 104, L031501 (2021).

[6] F. Cougoulic, Y. V. Kovchegov, A. Tarasov, and Y. Tawabutr, Quark and Gluon Helicity Evolution at Small x: Revised and Updated, J. High Energy Phys. 2022, 95 (2022).

[7] D. Adamiak et al., Global analysis of polarized DIS and SIDIS data with improved small-x helicity evolution, Phys. Rev. D. 108 (2023)

[8] R. Abdul Khalek et al., Science Requirements and Detector Concepts for the Electron-Ion Collider: EIC Yellow Report, Nucl. Phys. A 1026 (2022)

Extras

- **KPS-CTT Evolution Equations**

$$Q_f(x_{10}^2, zs) \sim Q_f^{(0)}(x_{10}^2, zs) + \iint \left[2\tilde{G} + \tilde{\Gamma} + Q_f - \bar{\Gamma}_f + 2\Gamma_2 + 2G_2\right] + \iint \left[Q_f + 2G_2\right]$$

$$\bar{\Gamma}_f(x_{10}^2, x_{21}^2, z's) \sim Q_f^{(0)}(x_{10}^2, z's) + \iint \left[2\tilde{G} + 2\tilde{\Gamma} + Q_f - \bar{\Gamma}_f + 2\Gamma_2 + 2G_2\right] + \iint \left[Q_f + 2G_2\right]$$

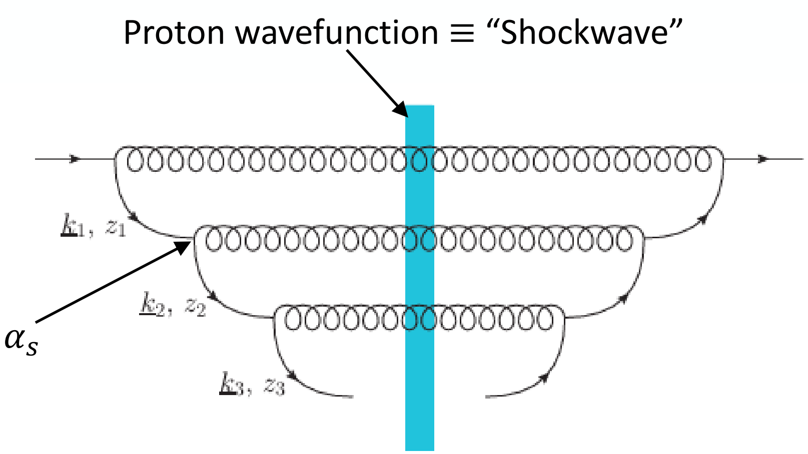
$$\tilde{G}(x_{10}^2, zs) \sim \tilde{G}^{(0)}(x_{10}^2, zs) + \iint \left[3\tilde{G} + \tilde{\Gamma} + 2G_2 + \Gamma_2 - \bar{\Gamma}_f\right] - \iint \left[Q_f + 2G_2\right]$$

$$\tilde{\Gamma}(x_{10}^2, x_{21}^2, z's) \sim \tilde{G}^{(0)}(x_{10}^2, z's) + \iint \left[3\tilde{G} + \tilde{\Gamma} + 2G_2 + \Gamma_2 - \bar{\Gamma}_f\right] - \iint \left[Q_f + 2G_2\right]$$

$$G_2(x_{10}^2, zs) \sim G_2^{(0)}(x_{10}^2, zs) + \iint \left[\tilde{G} + 2G_2\right]$$

$$\Gamma_2(x_{10}^2, x_{21}^2, z's) \sim G_2^{(0)}(x_{10}^2, z's) + \iint \left[\tilde{G} + 2G_2\right]$$

The two integrals are over the variables x_{10} and zs, which are related to the dipole's transverse size and momentum fraction.



Lifetime ordering provides two logs.

$$\left.\begin{array}{l} 1 \gg z_1 \gg z_2 \gg z_3 \gg \ldots \\ \dfrac{\underline{k}_1^2}{z_1} \ll \dfrac{\underline{k}_2^2}{z_2} \ll \dfrac{\underline{k}_3^2}{z_3} \ll \ldots \end{array}\right\} \int_{\frac{1}{s}}^{z} \frac{dz'}{z'} \int_{\frac{1}{s}}^{x_\perp} \frac{dx'_\perp}{x'_\perp} \left\{\begin{array}{l} \ln \dfrac{1}{x} \\ \ln \dfrac{1}{x} \end{array}\right.$$

$$\boldsymbol{\alpha_s \ll 1, \quad \ln \frac{1}{x} > 1 \quad \Rightarrow \quad \alpha_s \ln^2 \frac{1}{x} \approx 1}$$

Extras

- **Dipole Amplitudes as Correlations of Wilson Lines:**

$$G_{10}^{j}(zs) = \frac{1}{2N_c}\left\langle\left\langle \mathrm{tr}\left[V_{\underline{0}}^{\dagger} V_{\underline{1}}^{j\,G[2]} + \left(V_{\underline{1}}^{j\,G[2]}\right)^{\dagger} V_{\underline{0}}\right]\right\rangle\right\rangle(zs)$$

- **Cuts on Data**
  - We fit to all available DIS and SIDIS data within the kinematic ranges $5 \times 10^{-3} < x < 0.1$ and $1.69\ \mathrm{GeV}^2 < Q^2 < 10.4\ \mathrm{GeV}^2$
  - This leaves a total of 226 data points in the JAM database.

Data set (A_1)	**Target**	N_{pts}	χ^2/N_{pts}
SLAC (E142) [141]	^{3}He	1	0.60
EMC [146]	p	5	0.20
SMC [147, 149]	p	6	1.29
	p	6	0.53
	d	6	0.67
	d	6	2.26
COMPASS [150]	p	5	1.02
COMPASS [151]	p	17	0.74
COMPASS [152]	d	5	0.88
HERMES [153]	n	2	0.73
Total		59	0.91

Data set ($A_\parallel$)	**Target**	N_{pts}	χ^2/N_{pts}
SLAC(E155) [144]	p	16	1.28
	d	16	1.62
SLAC (E143) [143]	p	9	0.56
	d	9	0.92
SLAC (E154) [142]	^{3}He	5	1.09
HERMES [154]	p	4	1.54
	d	4	0.98
Total		63	1.19

Dataset (A_1^h)	**Target**	**Tagged Hadron**	N_{pts}	χ^2/N_{pts}
SMC [148]	p	h^+	7	1.03
	p	h^-	7	1.45
	d	h^+	7	0.82
	d	h^-	7	1.49
HERMES [158]	p	π^+	2	2.39
	p	π^-	2	0.01
	p	h^+	2	0.79
	p	h^-	2	0.05
	d	π^+	2	0.47
	d	π^-	2	1.40
	d	h^+	2	2.84
	d	h^-	2	1.22
	d	K^+	2	1.81
	d	K^-	2	0.27
	d	$K^+ + K^-$	2	0.97
HERMES [159]	^{3}He	h^+	2	0.49
	^{3}He	h^-	2	0.29
COMPASS [156]	p	π^+	5	1.88
	p	π^-	5	1.10
	p	K^+	5	0.42
	p	K^-	5	0.31
COMPASS [157]	d	π^+	5	0.50
	d	π^-	5	0.78
	d	h^+	5	0.90
	d	h^-	5	0.86
	d	K^+	5	1.50
	d	K^-	5	0.78
Total			104	1.01

Extras

- **Global Analysis:** Since the evolution equations are **closed** and **linear**, the full evolution can be computed from any single initial condition.

5 Flavor Singlet Initial Conditions:

$$Q_u^{(0)}, Q_d^{(0)}, Q_s^{(0)}, \tilde{G}^{(0)}, G_2^{(0)}.$$

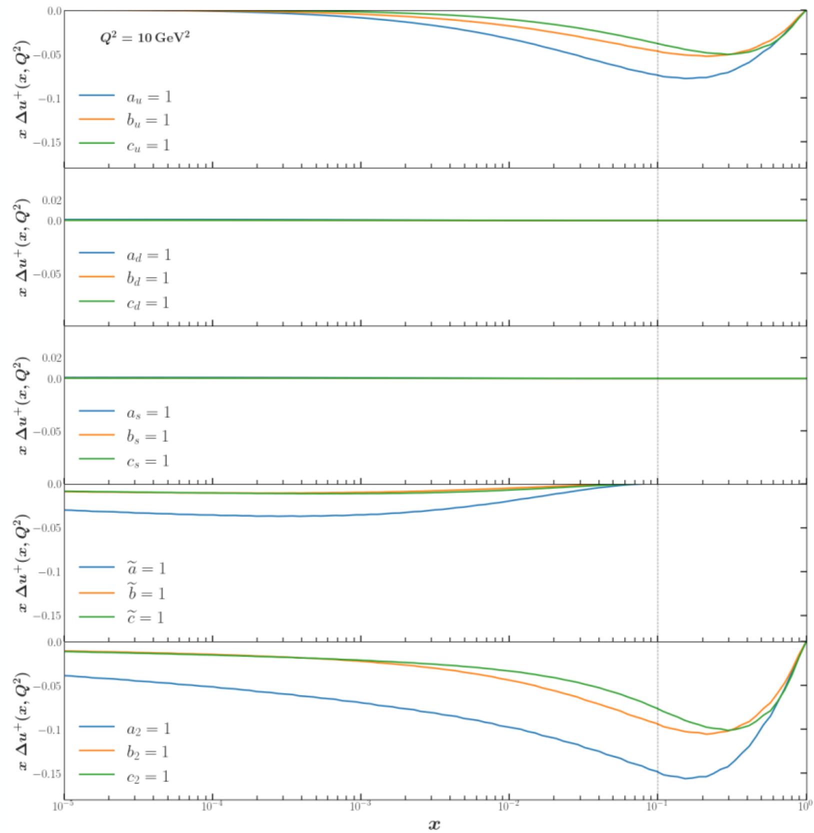
3 Flavor Non-singlet Initial Conditions:

$$Q_u^{NS\,(0)}, Q_d^{NS\,(0)}, Q_s^{NS\,(0)}$$

At Born-Level each initial condition is linearly dependent on logarithmic variables and can be parameterized as such:

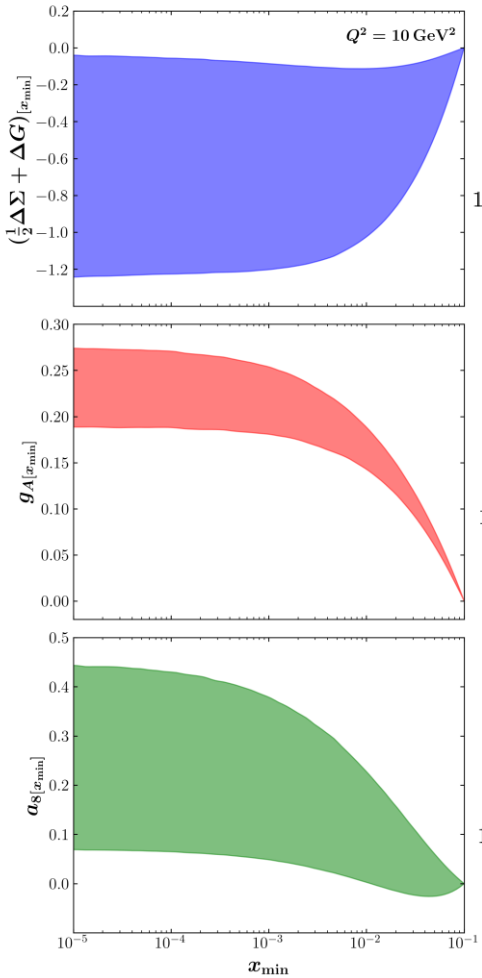
$$\eta \sim \ln\frac{zs}{\Lambda^2}, \quad s_{10} \sim \ln\frac{1}{x_{10}^2\Lambda^2}$$

$$\boldsymbol{G^{(0)} \approx a\eta + bs_{10} + c}$$



Extras

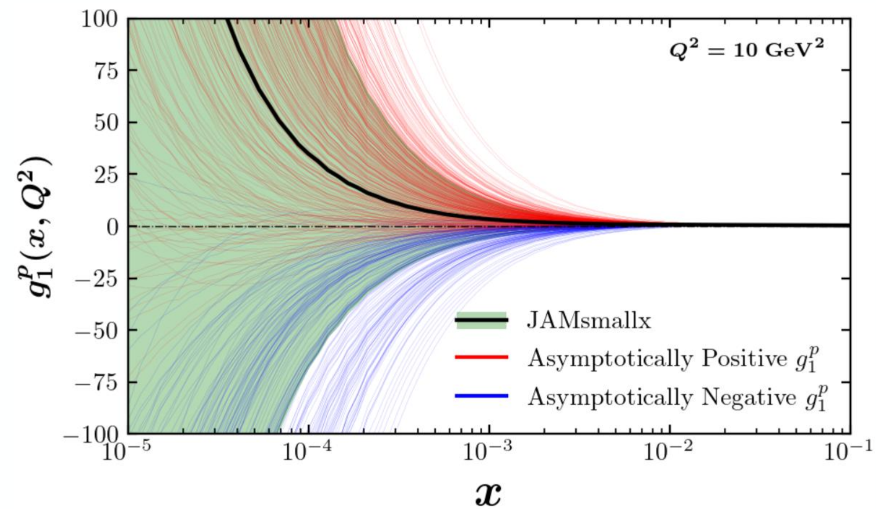
- **Small-x Extractions and Predictions:** Net spin, triplet g_A, octet g_8, and g_1^p.



$$\int_{10^{-5}}^{0.1} \mathrm{d}x \left(\frac{1}{2}\Delta\Sigma + \Delta G\right)(x) = -0.64 \pm 0.60$$

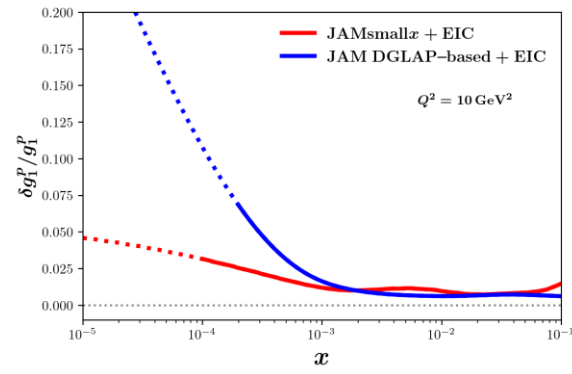
$$\int_{10^{-5}}^{0.1} \mathrm{d}x\, g_A(x) = 0.23 \pm 0.04$$

$$\int_{10^{-5}}^{0.1} \mathrm{d}x\, a_8(x) = 0.26 \pm 0.19$$



Extras

- **EIC Pseudo data**: Using small-x theory as an input, we can generate data consistent with the EIC's yellow report.
  - Kinematics: $10^{-4} < x < 10^{-1}$, $1.69\ \mathrm{GeV}^2 < Q^2 < 50\ \mathrm{GeV}^2$
  - Proton DIS: $\sqrt{s} = \{29, 45\ , 63, 141\}$ GeV, $100\ \mathrm{fb}^{-1}$ Luminosity
  - Deuteron/$^3\mathrm{He}$ DIS: $\sqrt{s} = \{29, 66, 89\}$ GeV, $10\ \mathrm{fb}^{-1}$ Luminosity
  - 2% point-to-point systematic uncertainty



- **Asymptotic Bimodality and $\widetilde{G}$**