

JOINT DARK ENERGY MISSION FIGURE OF MERIT SCIENCE WORKING GROUP

HEPAP

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Gary Bernstein,
On behalf of Rocky Kolb,
on behalf of the Science Working Group

FoMSWG

***Ad Hoc* JDEM Science Working Group Membership**

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JOINT DARK ENERGY MISSION SCIENCE WORKING GROUP STATEMENT OF TASK June 2008

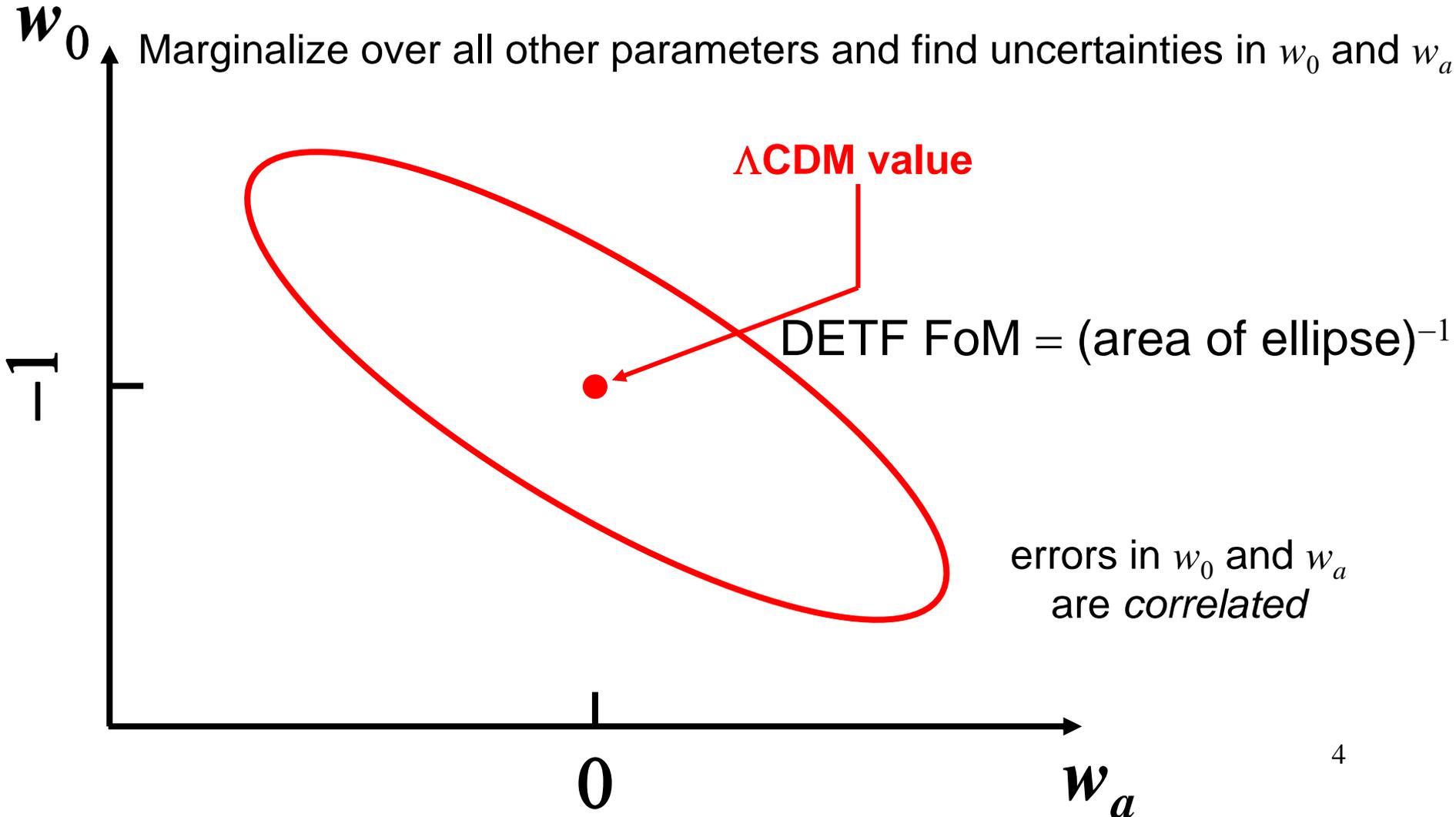
The purpose of this SWG is to continue the work of the Dark Energy Task Force in developing a quantitative measure of the power of any given experiment to advance our knowledge about the nature of dark energy. The measure may be in the form of a “Figure of Merit” (FoM) or an alternative formulation.

DETF FoM

$$\rho_{\text{DE}} = \rho_{\text{DE}}(\text{today}) \exp \left\{ -3 \int [1 + w(a)] d \ln a \right\} \quad \Lambda\text{CDM: } w(a) = -1$$

$$w(a) = w_0 + w_a(1 - a) \quad w = w_0 \text{ today } \& \ w = w_0 + w_a \text{ in the far past}$$

Marginalize over all other parameters and find uncertainties in w_0 and w_a



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Meetings: Washington 23-24 July
 Chicago 13-14 August

Phone conferences: In double digits

•FoMSWG Products:

- Quantitative evaluation process & formulae (complete, described herein)
- Prose containing qualitative statements that are of equal importance (in draft).
 - Importance of presenting systematic error analyses
 - Finite precision of FoM analyses
 - Scientific robustness as well as technical.

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From DETF:

The *figure of merit is a quantitative guide*: since the nature of dark energy is poorly understood, *no single figure of merit is appropriate* for every eventuality.

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FoMSWG (like DETF) adopted a [Fisher \(Information\) Matrix](#) approach toward assessing advances in dark energy science.

Reminder: the Fisher Matrix is the multidimensional version of $1/\sigma^2$:

- Describes the error ellipsoid of an experiment.
- Bigger is better
- Can be summed over experiments and priors to evaluate total constraint.

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1. Pick a fiducial cosmological model.

Not much controversy: Λ CDM [assumes Einstein gravity (GR)].

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2. Specify cosmological parameters of fiducial cosmological model (including parameterization of dark energy).

Not much controversy in non-dark energy parameters (we use WMAP5).

Parameterize dark energy as a function of redshift or scale factor

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2. Specify cosmological parameters of fiducial cosmological model (including parameterization of dark energy).

Issue #1: parameterization of $w(a)$

(want to know a function—but can only measure parameters)

- DETF: $w(a) = w_0 + w_a(1 - a)$ $w = w_0$ today & $w = w_0 + w_a$ in the far past
 - advantage: (only) two parameters
 - disadvantages: can't capture more complicated behaviors of w
 - FoM based on excluding $w \neq -1$ (either $w_0 \neq -1$ or $w_a \neq 0$)
- FoMSWG: $w(a)$ described by 36 piecewise constant values w_i defined in bins between $a = 1$ and $a = 0.1$
 - advantage: can capture more complicated behaviors
 - disadvantage: 36 parameters (issue for presentation, not computation)
 - merit based on excluding $w \neq -1$ (any $w_i \neq -1$)

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2. Specify cosmological parameters of fiducial cosmological model (including parameterization of dark energy).

Issue #2: parameterization of growth of structure (testing gravity)

- DETF discussed importance of growth of structure, but offered no measure
- Many (bad) ideas on how to go beyond Einstein gravity—no community consensus on clean universal parameter to test for modification of gravity
- FoMSWG made a choice, intended to be representative of the trends

$$\frac{d \ln G}{d \ln a} = [\Omega_m(z)]^\gamma; \quad G = G_0 e^{\int \Omega_m^\gamma d \ln a}$$

Growth of Structure = Growth of Structure (GR) + $\Delta\gamma \ln \Omega_M(z)$

**$\Delta\gamma$: one-parameter measure of
departure from Einstein gravity**

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3. For pre-JDEM and for a JDEM, produce “data models” including systematic errors, priors, nuisance parameters, etc.
 - Most time-consuming, uncertain, **controversial**, and critical aspect
 - Have to predict* “pre-JDEM” (circa 2016) knowledge of cosmological parameters, dark energy parameters, prior information, and nuisance parameters
 - Have to predict how a JDEM mission will perform
 - Depends on systematics that are not yet understood or completely quantified

**We made “best guess” for pre-JDEM
Strongly recommend don't reopen this can of worms**

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4. Predict how well JDEM will do in constraining dark energy.

This is what a Fisher matrix was designed to do:

- can easily combine techniques
- tool (blunt instrument?) for optimization and comparison

Technical issues, but fairly straightforward

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5. Quantify this information into a “figure of merit”

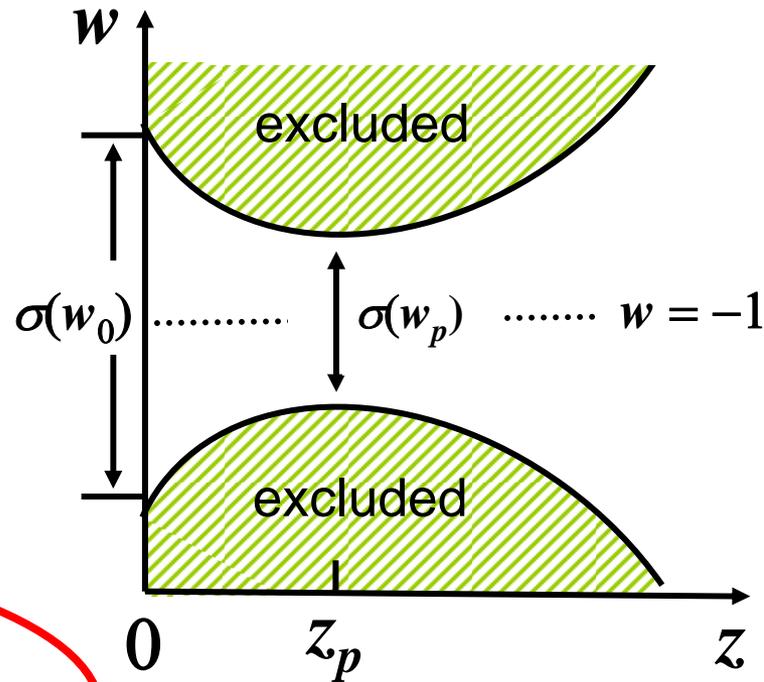
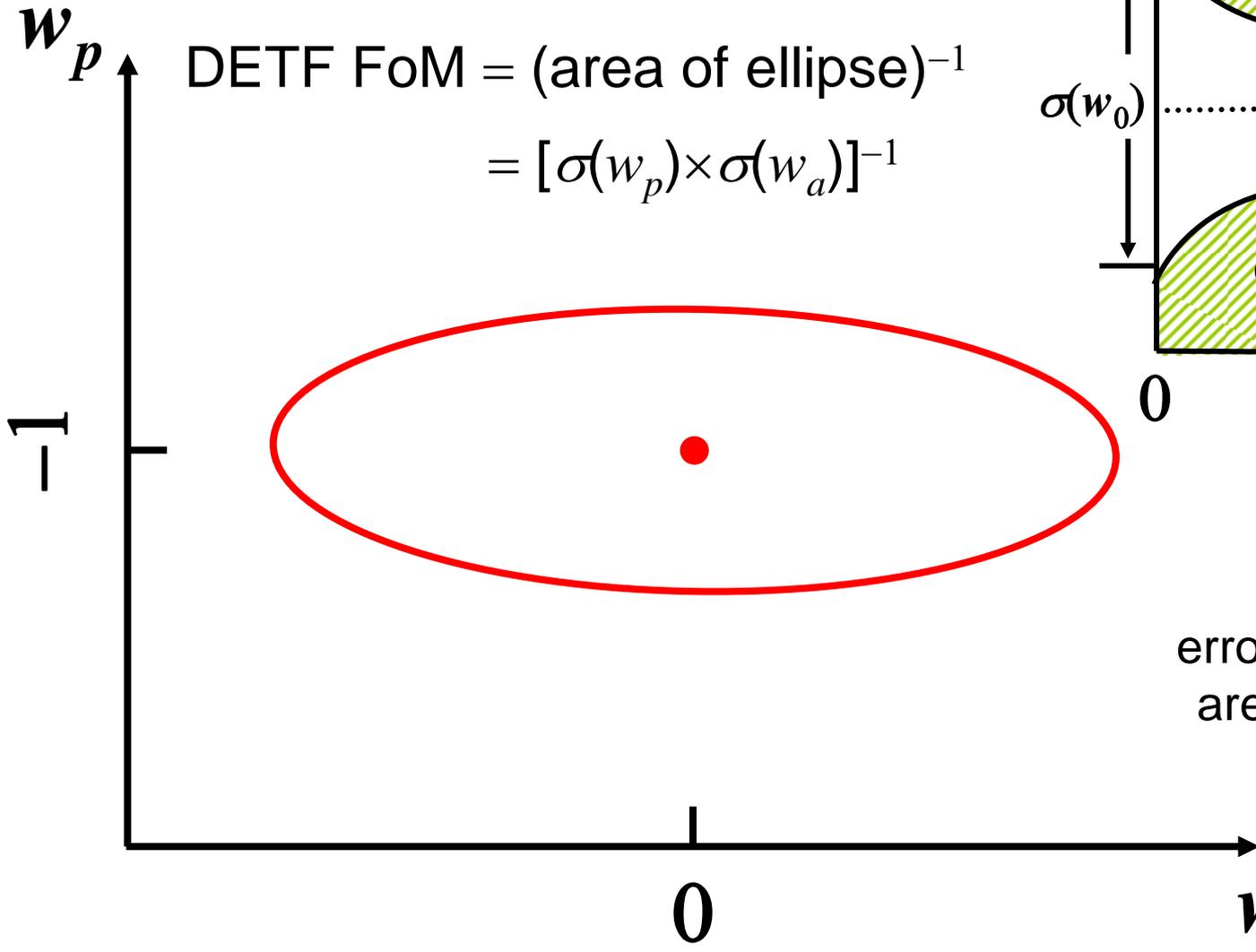
Discuss DETF figure of merit

Discuss where FoMSWG differs

DETF FoM

$$(w_0, w_a) \rightarrow (w_p, w_a)$$

$$\text{DETF FoM} = (\text{area of ellipse})^{-1} \\ = [\sigma(w_p) \times \sigma(w_a)]^{-1}$$



errors in w_p and w_a
are *uncorrelated*

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“... no single figure of merit is appropriate ...”

... but a couple of graphs and a few numbers can convey a lot!

- I. Determine the effect of dark energy on the expansion history of the universe by determining $w(a)$, parametrized as described above (higher priority)
- II. Determine the departure of the growth of structure from the result of the fiducial model to probe dark energy and test gravity
- III. A proposal should be free to argue for their own figure of merit

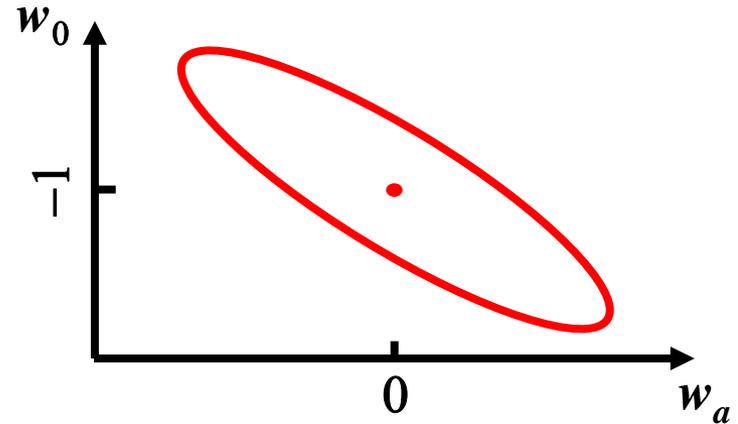
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I. Determine the effect of dark energy on the expansion history of the universe by determining $w(a)$, parametrized as described above (higher priority)

1. Assume growth of structure described by GR
2. Marginalize over all non- w “nuisance” parameters (e.g., H_0)
3. Perform “Principal Component Analysis” of $w(a)$
4. Then assume simple parameterization $w(a) = w_0 + w_a (1 - a)$ and calculate $\sigma(w_p)$, $\sigma(w_a)$, and z_p

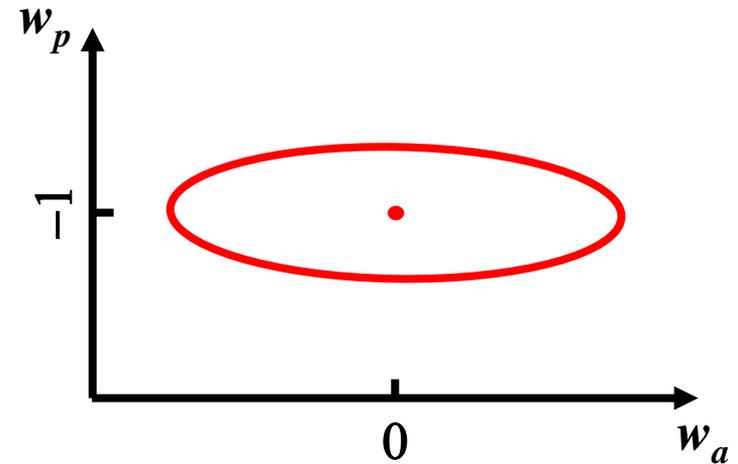
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- Generally, errors in different w_i are correlated (like errors in w_0 and w_a)



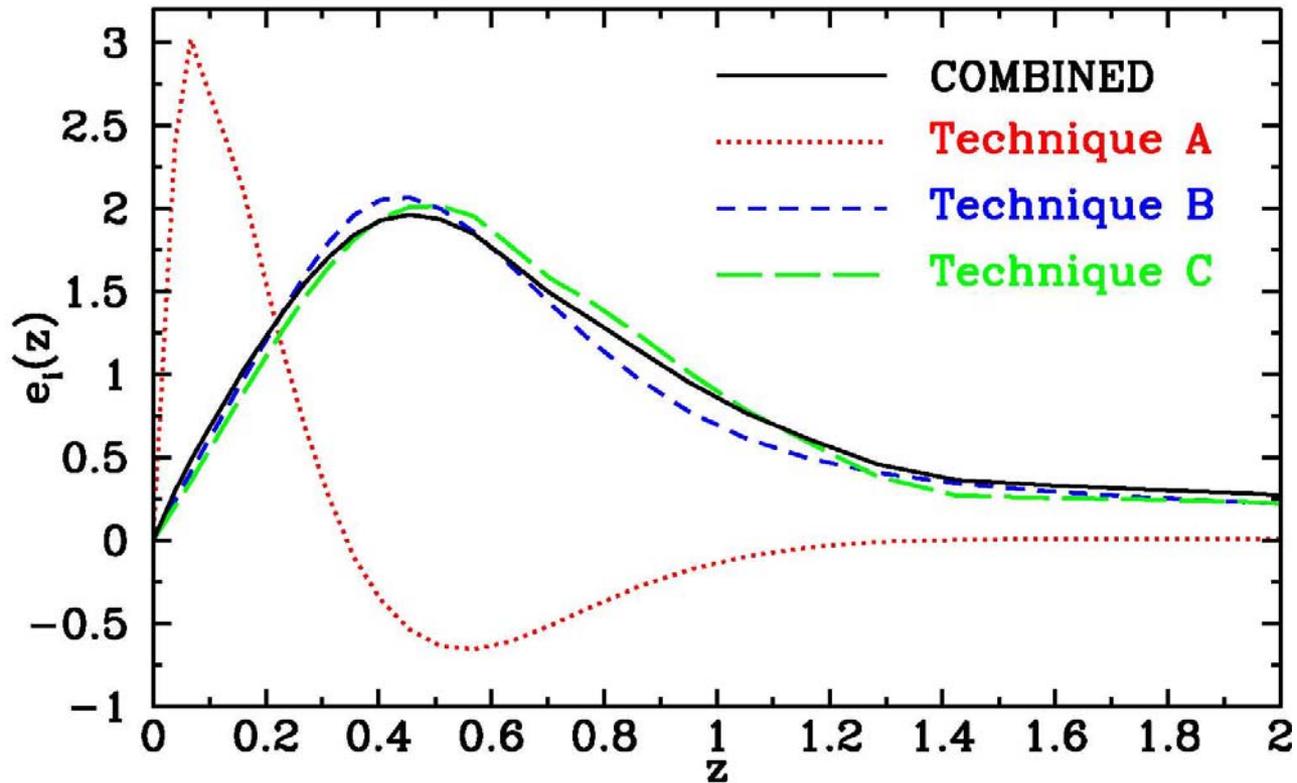
- Expand $w(a)$ in a complete set of orthogonal eigenvectors $e_i(a)$ with eigenvalues α_i (like w_p and w_a)

$$1 + w(a) = \sum_{i=0}^{35} \alpha_i e_i(a)$$



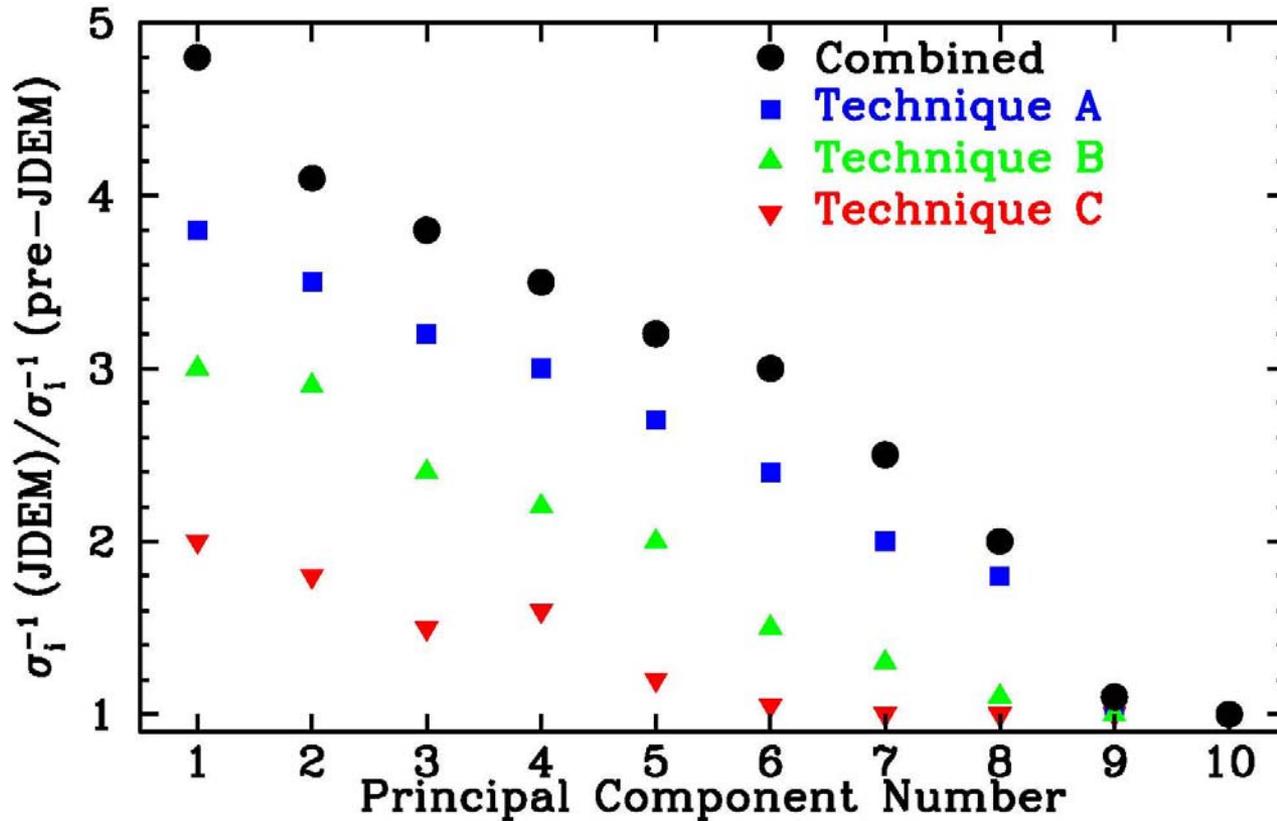
- Have 36 principal components
 - Errors $\sigma(\alpha_i)$ are uncorrelated
 - Rank how well principal components are measured
- Can do this for each technique individually & in combination

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- Graph of principal components as function of z informs on redshift sensitivity of technique [analogous to z_p] (may want first few PCs)
- Desirable to have reasonable redshift coverage
- Can visualize techniques independently and in combination

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- Graph of σ for various principal components informs on sensitivity to $w \neq -1$ [analogous to $\sigma(w_a)$ and $\sigma(w_p)$]
- If normalize to pre-JDEM, informs on JDEM improvement over pre-JDEM
- Again, can visualize techniques independently and in combination

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1. Assume growth of structure described by GR
 2. Marginalize over all non- w parameters
 3. Perform “Principal Component Analysis” of $w(a)$
 4. Then assume simple parameterization $w(a) = w_0 + w_a (1 - a)$
 5. Calculate $\sigma(w_p)$, $\sigma(w_a)$, and z_p
- } DETF analysis

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II. Determine the departure of the growth of structure from the result of the fiducial model to probe dark energy and test gravity

Calculate fully marginalized $\sigma(\Delta\gamma)$

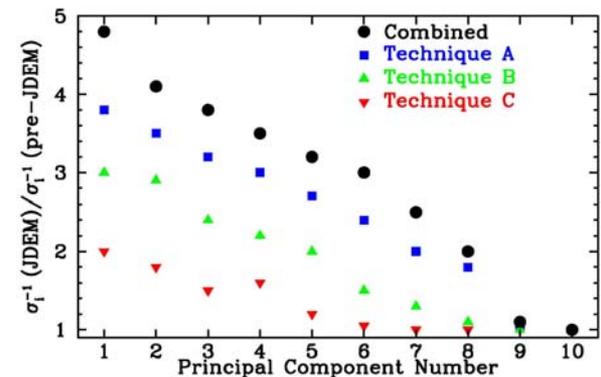
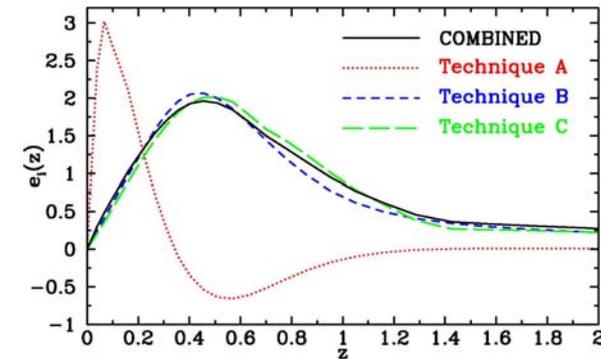
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III. A proposal should be free to argue for their own figure of merit

Different proposals will emphasize different methods, redshift ranges, and aspects of complementarity with external data. There is no unique weighting of these differences. Proposers should have the opportunity to frame their approach quantitatively in a manner that they think is most compelling for the study of dark energy. Ultimately, the selection committee or project office will have to judge these science differences, along with all of the other factors (cost, risk, etc). The FoMSWG method will supply one consistent point of comparison for the proposals.

Judgment on ability of mission to determine departure of Dark Energy from Λ :

1. Graph of first few principal components for individual techniques and combination
 - Redshift coverage
 - Complementarity of techniques
2. Graph of how well can measure modes
 - Can easily compare to pre-JDEM (as good as data models)
 - Relative importance of techniques (trade offs)
3. Three numbers: $\sigma(w_p)$, $\sigma(w_a)$, and z_p
 - Consistency check
4. One number, $\sigma(\Delta\gamma)$



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The FoMSWG end game

We will provide

- a longish letter to Kovar/Morse without too many technical details
- a technical paper posted on the archives
 - prescriptive
 - community can exercise the formalism
 - pre-JDEM Fisher matrices
- provide Fisher matrices and software tools on a website

We are wrapping up

- technical details on data models and software
- discussion of “threshold” issue
- finishing the technical paper

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Conclusions:

1. Figure(s) of Merit should not be the sole (or even most important) criterion
 1. Systematics
 2. Redshift coverage
 3. Departure from $w = -1$ must be convincing!
 4. Ability to differentiate “true” dark energy from modified gravity is important
 5. Multiple techniques important
 6. Robustness
2. Crucial to have common fiducial model and priors
3. Fisher matrix is the tool of choice
 1. FoMSWG (and DETF) put enormous time & effort into data models
 2. Data models can not be constructed with high degree of certainty
 3. Fisher matrix good for comparing and optimizing techniques
 4. Principal component analysis yields a lot of information
 5. We find a prescription for analysis and presentation
4. No one FoM gives complete picture