

# The Lattice QCD Project

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for the USQCD Collaboration

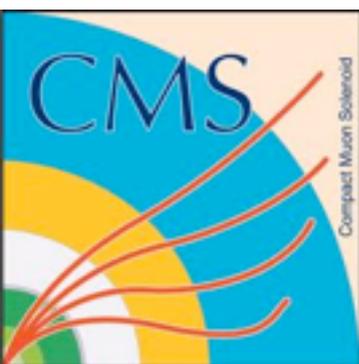
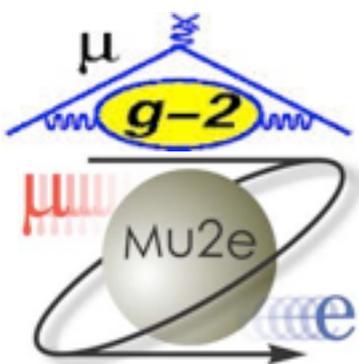
[www.usqcd.org](http://www.usqcd.org)

HEPAP  
Sept. 5-6, 2013



# Lattice field theory for high-energy physics

- Lattice gauge theory plays a critical role in the HEP search for new physics at the Energy and Intensity Frontiers.
- In the past decade, as error budgets became fully quantitative, **lattice QCD** played a critical role in the success of the **quark-flavor factories**.
- In the coming decade, **high-precision lattice gauge theory calculations** are needed *throughout* the HEP experimental program, e.g.:
  - **g-2**, **Mu2e**, **LBNE**, **Nova**, **CDMS**, **Belle-2**, **LHCb**, **CMS**, **Atlas**, **ILC**.
- The USQCD planned scientific program is aligned with the HEP experimental program discussed at Snowmass.
- We hope you will agree that facilities for numerical lattice QCD are an essential theoretical adjunct to the experimental HEP physics program.



# USQCD and the US lattice community

- USQCD comprises almost all US lattice gauge theorists (~150 people).
- USQCD purchases and deploys hardware and develops software infrastructure for lattice calculations in the US.
  - The hardware bought by the **LQCD project** is operated as a **facility** (like Fermilab during the fixed-target program).
  - The smaller **physics collaborations** which make up USQCD **compete** for time on these resources for their physics.
  - Resources are allocated by the USQCD Scientific Program Committee among physics projects to meet Collaboration goals outlined in Collaboration white papers (and Snowmass documents and the Project X Physics Book).



# USQCD computing activities



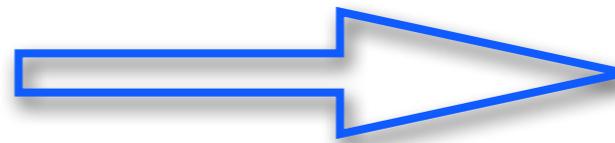
The Ds cluster at Fermilab

- Hardware
  - USQCD requires similar numbers of cycles on jobs of all sizes: 16-core, 100-core, 1,000-core, 10,000-core, and 100,000-core.
  - DoE Leadership Computing facilities at Argonne and Oak Ridge supply our requirements for jobs > 10,000-core.
  - We have an even larger need for cycles for jobs < 10,000-core. These are supplied predominantly by the **LQCD Project**, whose CD0 is under consideration.
    - Also apply at NERSC, XSEDE, Blue Waters, ...
  - DOE has supported USQCD through joint HEP/NP projects LQCD (FY06-FY09) and LQCD-ext (FY10-FY14).
  - These provide funds for dedicated capacity hardware at Fermilab, Jefferson Lab, and Brookhaven, and for support personnel.
- Software
  - USQCD has received grants for software development from the DOE's SciDAC Program since 2003. The most recent are SciDAC-3 grants from HEP and NP for about \$1 M each for creating lattice QCD software infrastructure: community **libraries**, **community codes**, **optimization** and **porting** to new architectures, implementation of up-to-the-minute **algorithm advances**...

# Anatomy of a typical lattice calculation



ALCF Blue Gene



TB file sizes



Fermilab Ds

Generate gauge configurations on a leadership facility or supercomputer center. Many tens of millions of BG/Q core-hours in a single job.

A single highly optimized program, very long single tasks, moderate I/O and data storage. Needs **high capability** computing.

Transfer to labs for analysis on clusters. Comparable CPU requirements.

Large, heterogeneous analysis code base, 10,000s of small, highly parallel tasks, heavy I/O and data storage. Needs **high capacity** computing.

Two comparably sized tasks with quite different hardware requirements.

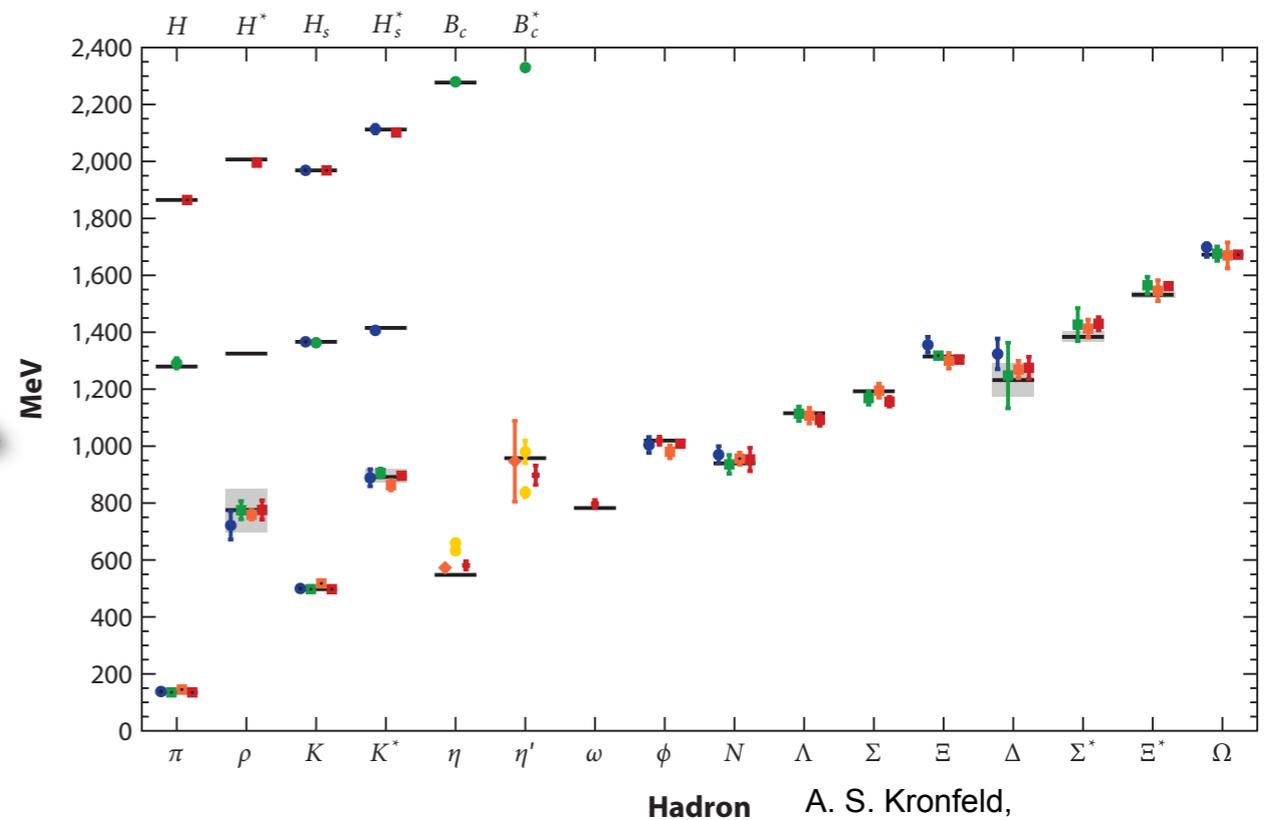
# USQCD and the broader HEP community

- USQCD participated actively in the [Snowmass](#) process.
  - Steve Gottlieb was a convener of the Computing Frontier section, and Ruth Van de Water and Tom Blum were conveners of the Lattice Field Theory subgroup.
  - We gave talks in both the Energy Frontier and Intensity Frontier sessions and contributed to writing several of the documents.
- Members of USQCD wrote a chapter on lattice field theory for the [Project X Physics Book](#).
- USQCD regularly organizes "[Lattice Meets Experiment](#)" [workshops](#) on such topics as:
  - Extreme QCD (2012); Beyond the Standard Model (2011 & 2012); Fluctuations, Correlations, and RHIC low energy runs (2011); Excited Hadronic States and the Deconfinement Transition (2011); Quark flavor physics (2010); ....
- Members of USQCD regularly speak at [experimental collaboration meetings](#) such as [g-2](#), [Belle](#), [BES](#), [ORKA](#), [HFAG-Vxb](#), ...



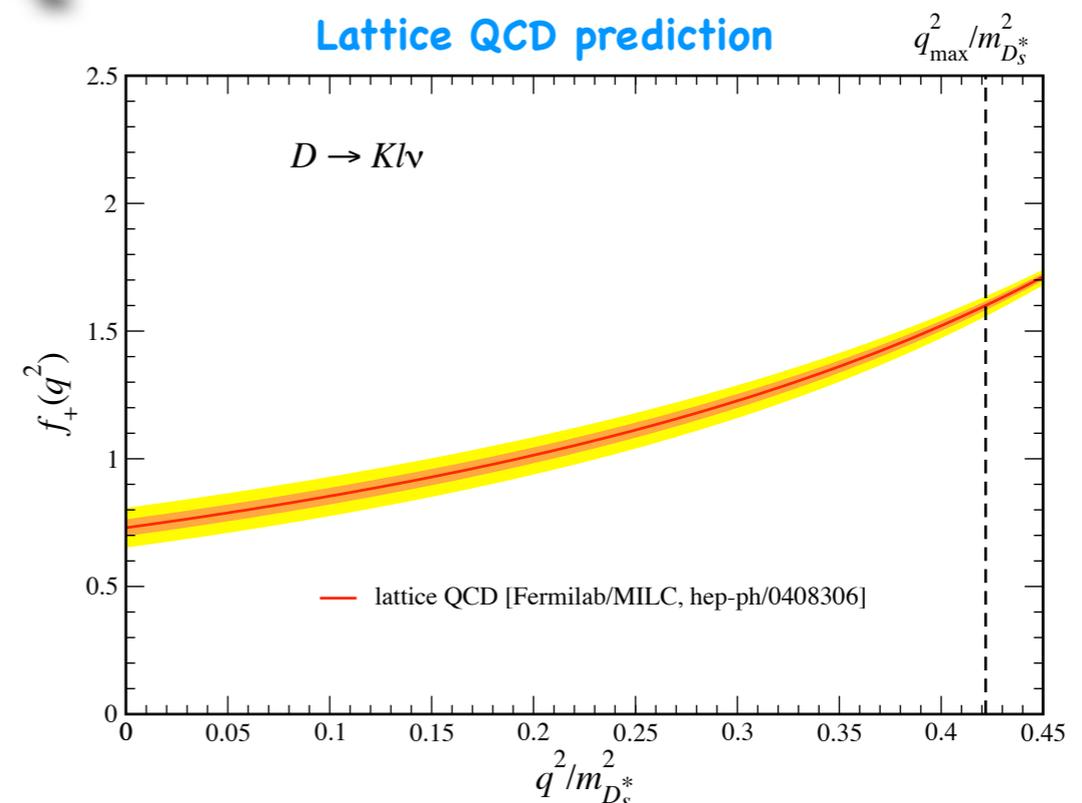
# Validation of lattice calculations

- Comparison with experiment, e.g. hadron spectrum to 1%.
- Prediction of quantities not known in advance, e.g. shape of the  $D \rightarrow K$  semileptonic form factor.
- Important calculations are checked by multiple groups using different methods, different discretizations, etc.
- Many internal checks: recovery of rotational invariance, recovery of Lorentz invariance, test of software using gauge invariance.



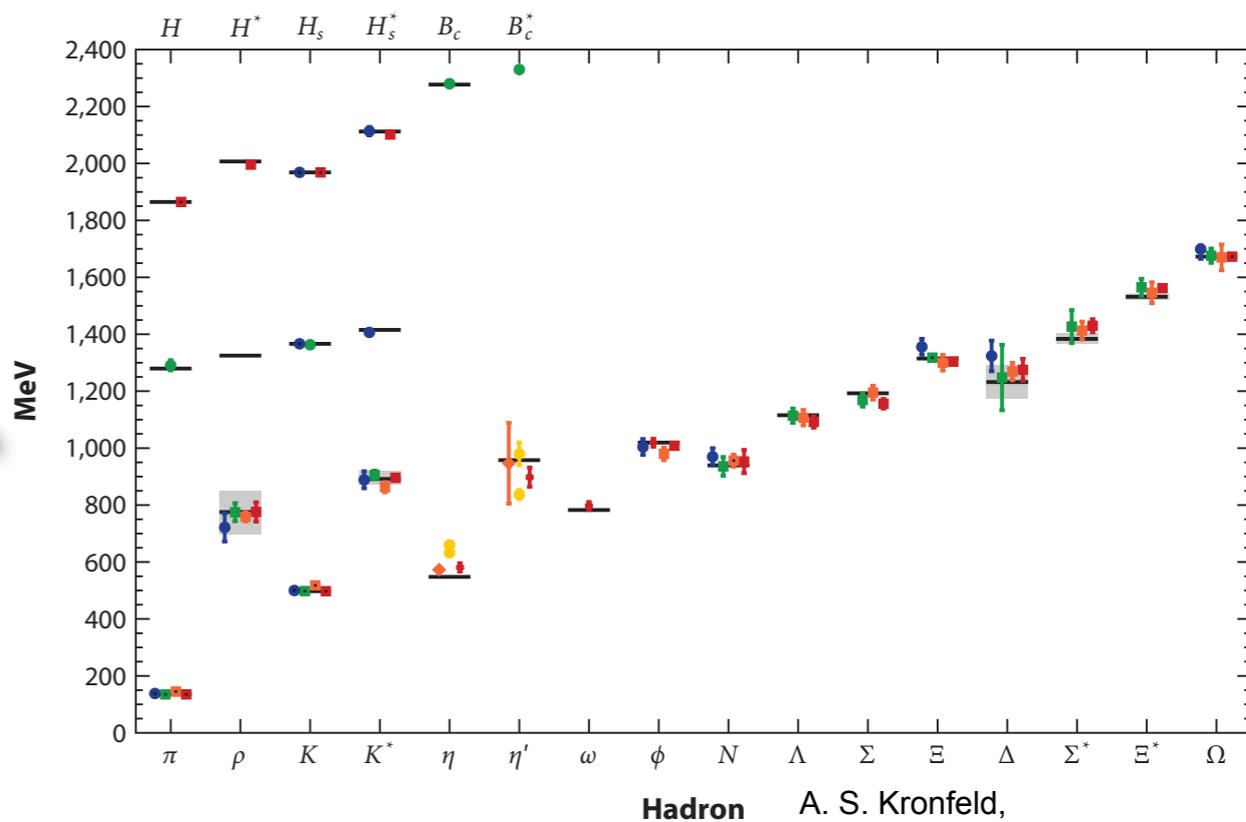
A. S. Kronfeld,  
Ann.Rev.Nucl.Part.Sci. 62 (2012) 265-284.

August 2004:  
Lattice QCD prediction



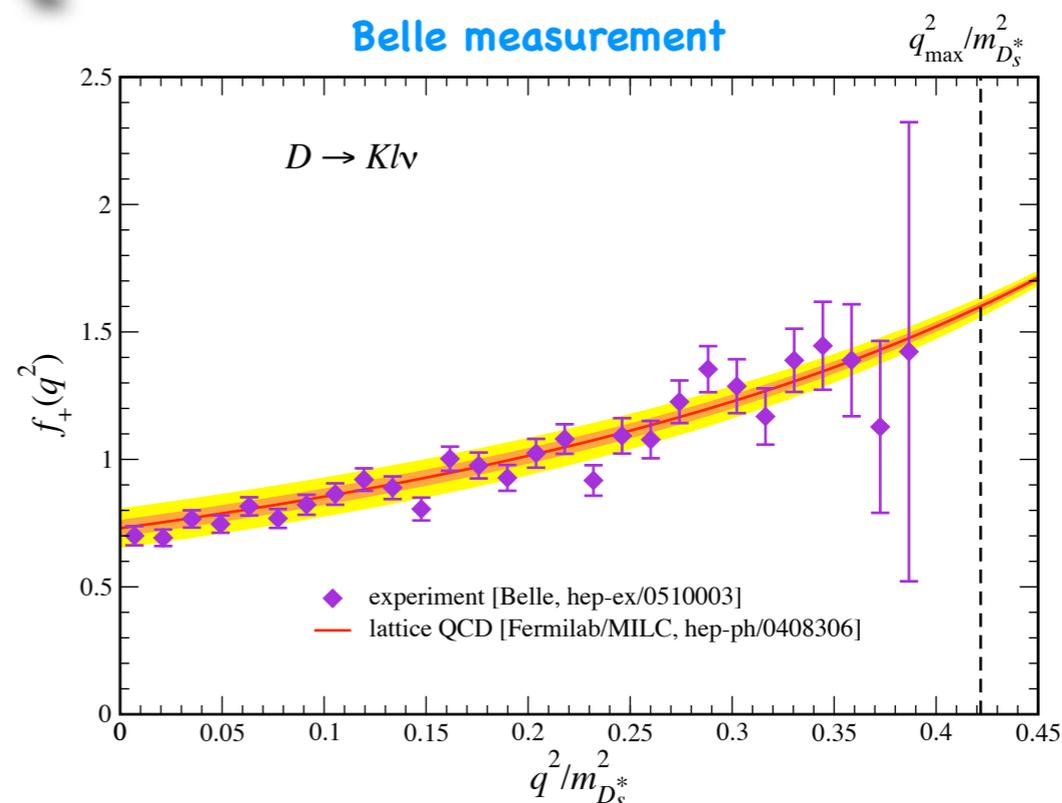
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A. S. Kronfeld,  
Ann.Rev.Nucl.Part.Sci. 62 (2012) 265-284.

October 2005:  
Belle measurement



# Precision measurements of Higgs properties

- **Bottom-quark mass** and  $\alpha_s$  contribute the largest uncertainties to the dominant Higgs decay channel and Higgs total width

**Table 1-5.** Uncertainties on  $M_H = 126$  GeV Standard Model widths arising from the parametric uncertainties on  $\alpha_s$ ,  $m_b$ , and  $m_c$  and from theory uncertainties.

Channel	$\Delta\alpha_s$	$\Delta m_b$	$\Delta m_c$	Theory Uncertainty	Total Uncertainty
$H \rightarrow \gamma\gamma$	0%	0%	0%	$\pm 1\%$	$\pm 1\%$
$H \rightarrow b\bar{b}$	$\pm 2.3\%$	$+3.3\%$ $-3.2\%$	0%	$\pm 2\%$	$\pm 6\%$
$H \rightarrow \tau^+\tau^-$	0%	0%	0%	$\pm 2\%$	$\pm 2\%$
$H \rightarrow WW^*$	0%	0%	0%	$\pm 0.5\%$	$\pm 0.5\%$
$H \rightarrow ZZ^*$	0%	0%	0%	$\pm 0.5\%$	$\pm 0.5\%$

[Snowmass Higgs WG Report](#), p 5.

*“Improvement in alphas and quark masses will come from lattice gauge theory.*

*These are necessary inputs to precision Higgs theory and other precision programs.”*

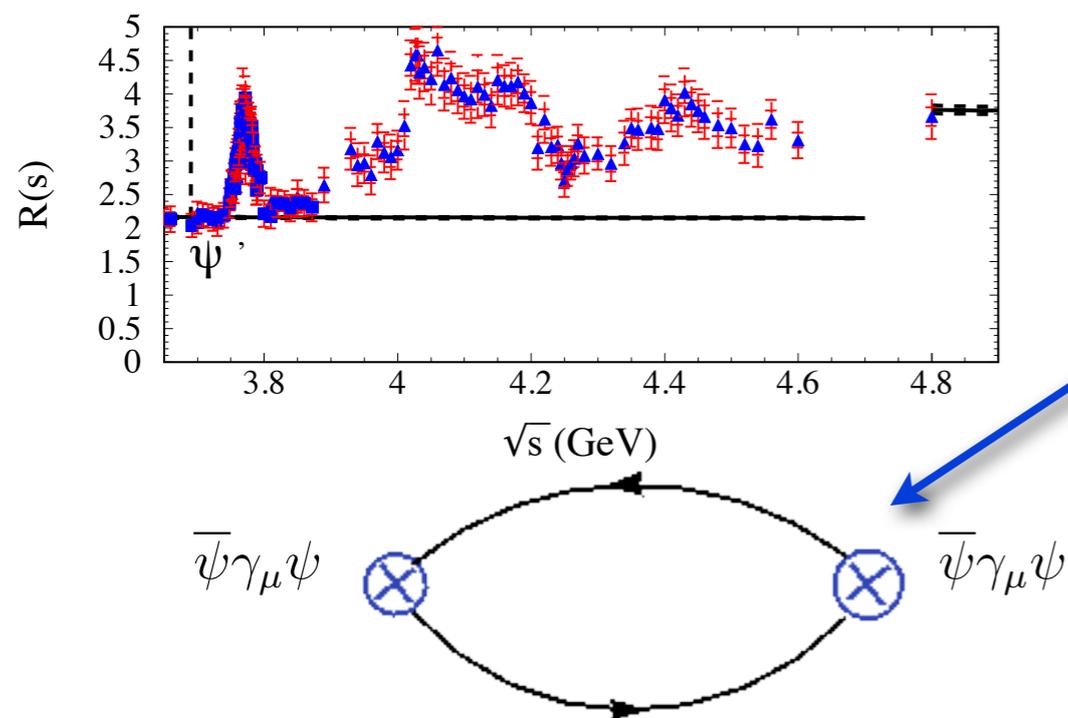
Brock/Peskin Snowmass [Energy Frontier summary talk](#), p118.

	Higgs X-section Working Group [34]	PDG[1]	Non-lattice	Lattice (2013)	Lattice (2018)	Prospects from ILC/TLEP/LHeC
$\delta\alpha_s$	0.002	0.0007	0.0012 [1]	0.0006 [24]	0.0004	0.0001–0.0006 [8, 27, 28]
$\delta m_c$ (GeV)	0.03	0.025	0.013 [31]	0.006 [24]	0.004	-
$\delta m_b$ (GeV)	0.06	0.03	0.016 [31]	0.023 [24]	0.011	-

[Snowmass QCD WG Report](#), p 10.

- An **ILC/TLEP/muon collider** goal: Higgs branching fractions to  $\sim 1\%$ ;
  - requires  $m_b$  to  $\sim 0.25\%$  not to affect the analysis.
  - Lattice QCD will do this.
  - Lattice QCD has already determined  $m_b$ ,  $m_c$ , and  $\alpha_s$  more precisely than is currently being assumed in discussions of Higgs decay channels.

# Quark masses and $\alpha_s$ from lattice QCD.



Moments of the heavy quark production cross section in  $e^+e^-$  annihilation can be related to the derivatives of the vacuum polarization to obtain the heavy quark masses. These can easily be obtained from lattice QCD instead of  $e^+e^-$ .

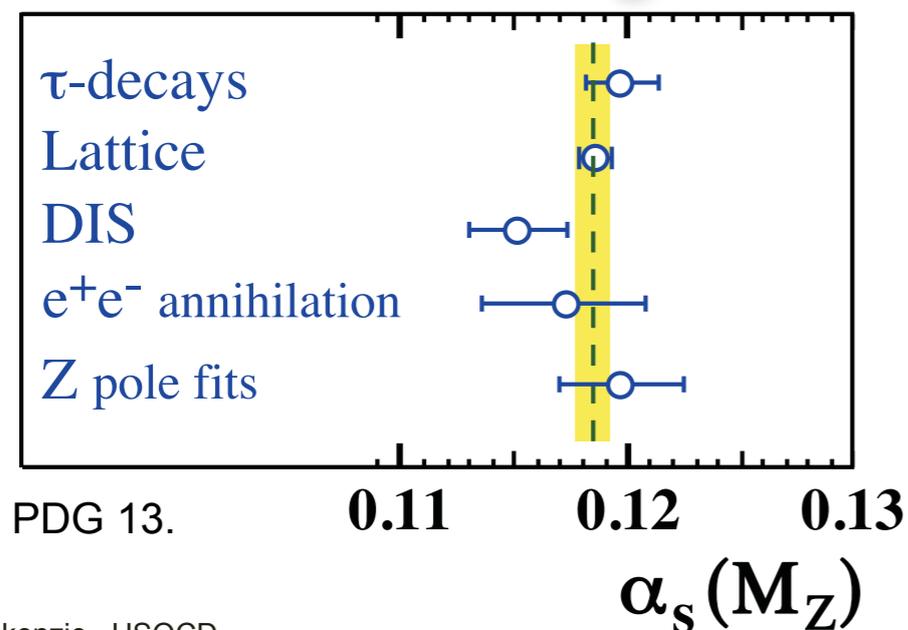
Can be calculated in perturbation theory. Known to  $O(\alpha_s^3)$  (Chetyrkin et. al.)

$$m_b(m_b, n_f = 5) = 4.164(23) \text{ GeV}$$

$$m_c(m_c, n_f = 4) = 1.273(6) \text{ GeV}$$

HPQCD, McNeile et al.

- Adler function ⊖ JLQCD
- Schrödinger functional ⊖ PACS-CS
- Ghost-gluon vertex ⊖ ETM
- QQbar correlators ⊖ HPQCD
- Wilson loops ⊖ HPQCD



World  $\alpha_s$  results are very robust: remove the two most precise lattice results and the remaining lattice results give the same answer with slightly increased errors; remove all lattice results and the remaining results give the same answer with a little more increased errors.

PDG 13 world average:

$$\alpha_s(M_Z^2) = 0.1184 \pm 0.0007$$

$$\alpha_s(M_Z^2) = 0.1183 \pm 0.0012 \text{ (w/o lattice results)}$$

# Lattice-QCD constraints on the CKM matrix

- Cabibbo-Kobayashi-Maskawa matrix elements and phase are **fundamental parameters of the Standard Model that enter as parametric inputs to Standard Model predictions for many flavor-changing processes** such as neutral kaon mixing and  $K \rightarrow \pi \nu \nu$  decays.

- Simple matrix elements involving single particles enable determinations of all CKM matrix elements except  $|V_{tb}|$ .

\*Neutral kaon mixing can also be used to obtain the phase  $(\rho, \eta)$ .

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \pi \rightarrow \ell \nu & K \rightarrow \ell \nu & B \rightarrow \ell \nu \\ & K \rightarrow \pi \ell \nu & B \rightarrow \pi \ell \nu \\ V_{cd} & V_{cs} & V_{cb} \\ D \rightarrow \ell \nu & D_s \rightarrow \ell \nu & B \rightarrow D \ell \nu \\ D \rightarrow \pi \ell \nu & D \rightarrow K \ell \nu & B \rightarrow D^* \ell \nu \\ V_{td} & V_{ts} & V_{tb} \\ \langle B_d | \bar{B}_d \rangle & \langle B_s | \bar{B}_s \rangle & \end{pmatrix}$$

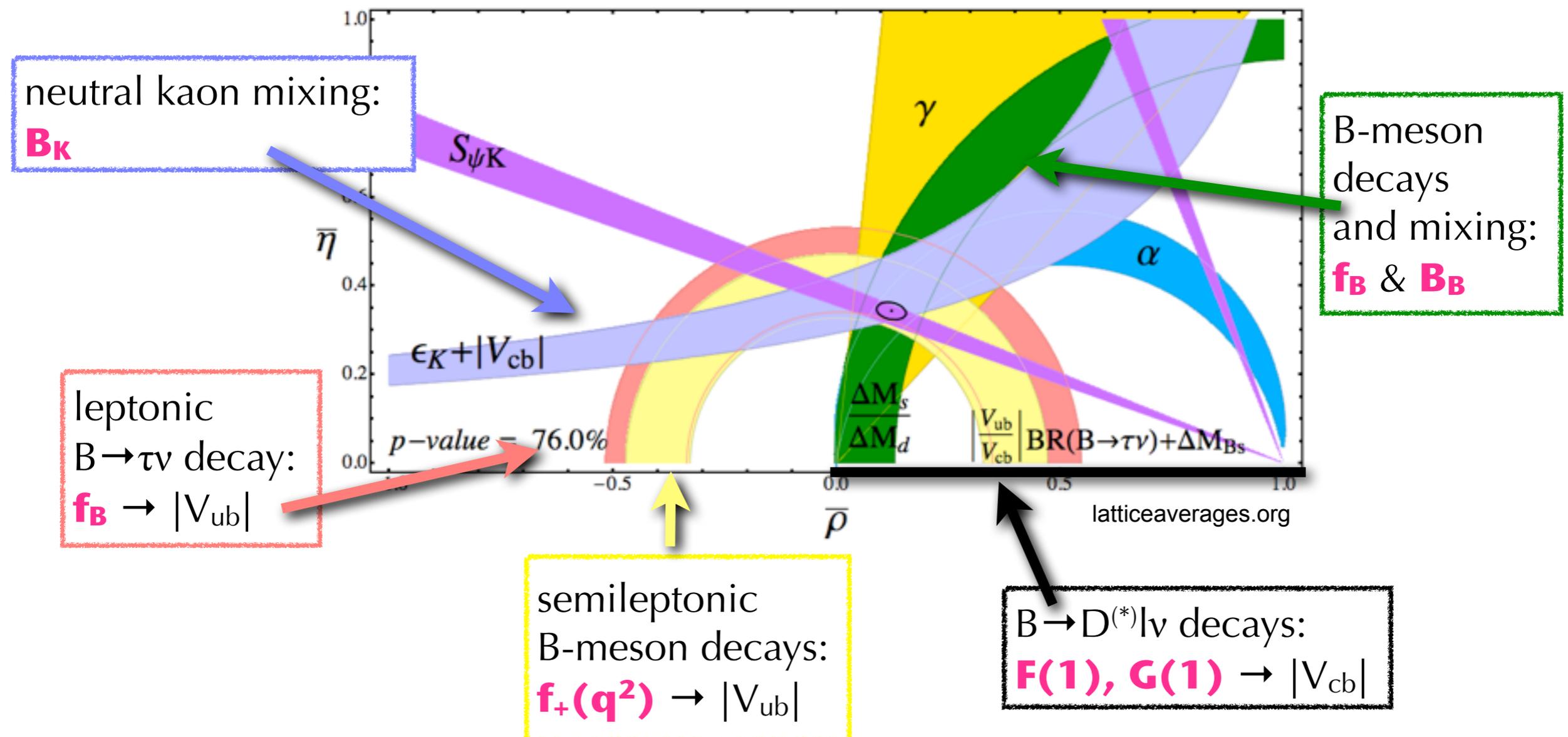
$$\text{(Experiment)} = (\text{known}) \times (\text{CKM factors}) \times (\text{Hadronic Matrix Element})$$

$$\epsilon_K, \Delta m_{(d,s)}, \frac{d\Gamma(B \rightarrow \pi \ell \nu)}{dq^2}, \frac{d\Gamma(B \rightarrow D^{(*)} \ell \nu)}{dw}, \dots$$

Absorb nonperturbative QCD effects into quantities such as decay constants, form factors, and bag-parameters that must be computed numerically with **LATTICE QCD**

# Lattice QCD inputs to the unitarity triangle

- Standard approach to search for new physics in the flavor sector is by overconstraining the angles and sides of the CKM unitarity triangle.
- Many constraints require lattice-QCD calculations of **hadronic weak matrix elements**.



- USQCD calculations lead the world on all quantities shown except  $B_K$ , for which we are competitive.

# Project goals: how are we doing?

- LQCD-ext proposal written in 2007-2008 showed a table of “present status and future prospects for lattice calculations which directly determine elements of the CKM matrix”.

Quantity	CKM element	present expt. error	present lattice error	2009 lattice error ( <i>prediction</i> )	2013 lattice error ( <i>actual</i> )
$f_K/f_\pi$	$V_{us}$	0.3%	0.9%	0.5%	0.4%
$f_{K\pi}(0)$	$V_{us}$	0.4%	0.5%	0.3%	0.4%
$D \rightarrow \pi l \nu$	$V_{cd}$	3%	11%	6%	4.4%
$D \rightarrow K l \nu$	$V_{cs}$	1%	11%	5%	2.5%
$B \rightarrow D^* l \nu$	$V_{cb}$	1.8%	2.4%	1.6%	1.8%
$B \rightarrow \pi l \nu$	$V_{ub}$	3.2%	14%	10%	8.7%

- Last column is new and shows the current errors in these same quantities.

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$f_{K\pi}(0)$					0.4%
$D \rightarrow \pi \ell \nu$					4.4%
$D \rightarrow K \ell \nu$	$V_{cs}$	1%	11%	5%	2.5%
$B \rightarrow D^* \ell \nu$	$V_{cb}$	1.8%	2.4%	1.6%	1.8%
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We are meeting our goals and are leading the world for all of the quantities shown

- Last column is new and shows the current errors in these same quantities.

# The US flavor effort

- USQCD is **leading the world** in precision calculations of hadronic matrix elements needed to obtain **CKM parameters** and test the Standard-Model CKM framework.
- The US quark flavor effort is **focusing more on the needs of experiment and phenomenology** than those of other parts of the world.
  - E.g., many interesting field theory calculations are being done with  $N_f = 2$ , or static  $b$  quarks, ..., not directly applicable to phenomenology. (USQCD's are with  $N_f=3$  or 4, and physical  $b$  quark masses.)
- For many quantities, especially in  $B$  physics, US calculations are not simply the best calculations, they are the only calculations.
  - $B \rightarrow D^{(*)} l \nu$  for  $V_{cb}$ ,
  - $B \rightarrow \pi l \nu$  for  $V_{ub}$ ,
  - $B$  mixing,  $B_s$  mixing.



# The LQCD-ext II Proposal

- USQCD has proposed an extension of our current hardware project hardware project for the period 2015 - 2019.
- In order to meet our scientific goals during this period, a mixture of access to the DOE's Leadership Class and acquisition of dedicated hardware is needed.
- The plan calls for sustaining around five sustained petaflop/s on each class of computer by its final year:

Fiscal Year	Dedicated Hardware (TF-Years)	Leadership Class Machines (TF-Years)
2015	325	430
2016	520	680
2017	800	1,080
2018	1,275	1,715
2019	1,900	2,720
Total	4,820	6,625



# LQCD-ext II Budget

- The proposed budget for LQCD-ext II is \$23.50 M over five years. Coming from both **HEP** and **NP**. The same as for LQCD-ext plus its ARRA-funded supplement.

Fiscal Year	Hardware Budget	Operations Budget	Total Budget
2015	2.63	1.95	4.58
2016	2.63	2.02	4.65
2017	2.63	2.07	4.70
2018	2.63	2.13	4.76
2019	2.63	2.18	4.81
Totals	13.15	10.36	23.50

TABLE IX: Proposed budget for LQCD III in millions of dollars.

- LQCD-ext II will allow the accomplishment of the scientific goals outlined in the LQCD-ext II proposal, the Snowmass Computing Frontier report, and the USQCD white papers.
- The following are some examples.

# Muon $g-2$



- Measured muon  $g-2$  disagrees with Standard-Model prediction by  $>3\sigma$ , and new Fermilab Muon  $g-2$  Experiment will reduce the experimental error to approximately 0.14 ppm.
- Dominant uncertainty in Standard-Model prediction for muon  $g-2$  from hadronic **light-by-light** and **vacuum polarization** contributions.
- Current estimate for uncertainty in light-by-light comes from the spread of models.

*“Finding a new approach [for the light-by-light contributions], such as lattice QCD, in which uncertainties are systematically improvable, is crucial for making greatest use of the next round of experiments.”*

Snowmass Charged Leptons WG Report, p 31.

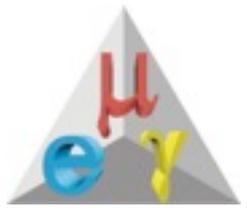
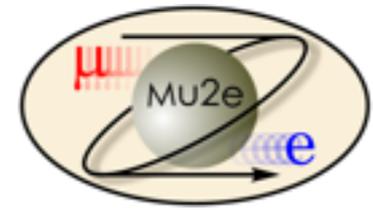
Contribution	Result ( $\times 10^{11}$ )		Error
QED (leptons)	116 584 718	$\pm 0.14 \pm 0.04_{\alpha}$	0.00 ppm
HVP(lo) [1]	6 923	$\pm 42$	0.36 ppm
HVP(ho)	-98	$\pm 0.9_{\text{exp}} \pm 0.3_{\text{rad}}$	0.01 ppm
HLbL [2]	105	$\pm 26$	0.22 ppm
EW	154	$\pm 2 \pm 1$	0.02 ppm
Total SM	116 591 802	$\pm 49$	0.42 ppm

Reducing **hadronic light by light** contribution to uncertainty requires new lattice methods. Promising early results have been obtained.

See Van de Water plenary Precision Frontier colloquium at Snowmass.

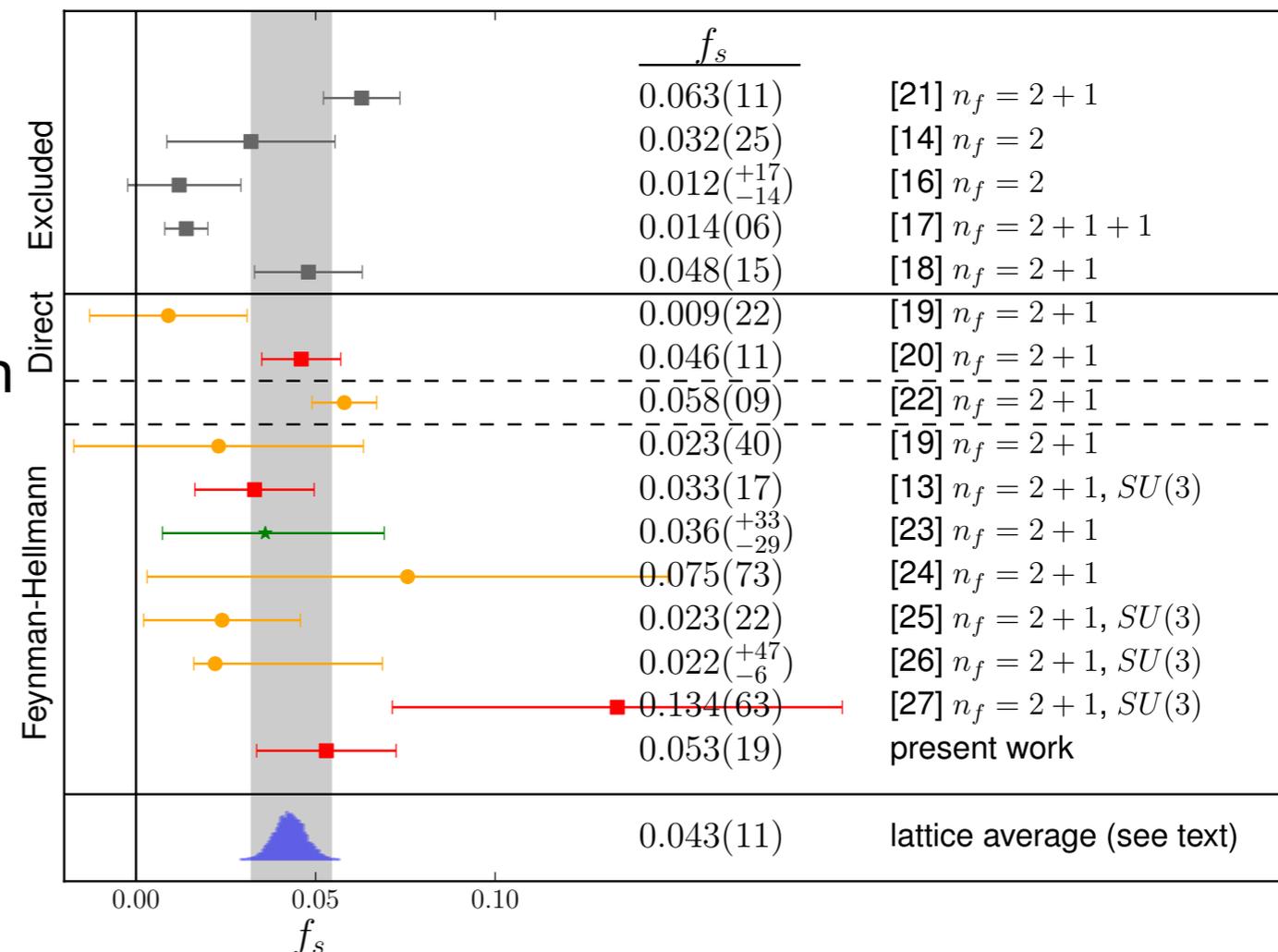


# $\mu$ to $e$ conversion (and dark matter)



- Observation of charged-lepton flavor violation at **Mu2e** would be unambiguous evidence of new physics.
  - Combining measured rates of  $\mu \rightarrow e \gamma$  and  $\mu \rightarrow e$  conversion on different target nuclei can distinguish between theories, but predictions for  $\mu \rightarrow e$  conversion rate depend upon light- and strange-quark contents of nucleon from lattice QCD.
- Same hadronic matrix elements also needed to interpret **dark-matter detection** experiments in which the DM particle scatters off a nucleus.
- Calculations improved significantly in recent years, and already rule out much larger values of  $f_s = m_s \langle N | s s | N \rangle / M_N$  favored by early non-lattice estimates.
- Even greater precision is needed for **Mu2e** and dark-matter searches.
  - Realistic goal for next five years is to pin down values with  $\sim 10\text{-}20\%$  errors.

Junnarkar & Walker-Loud, arXiv:1301.1114



# Neutrino Cross Section



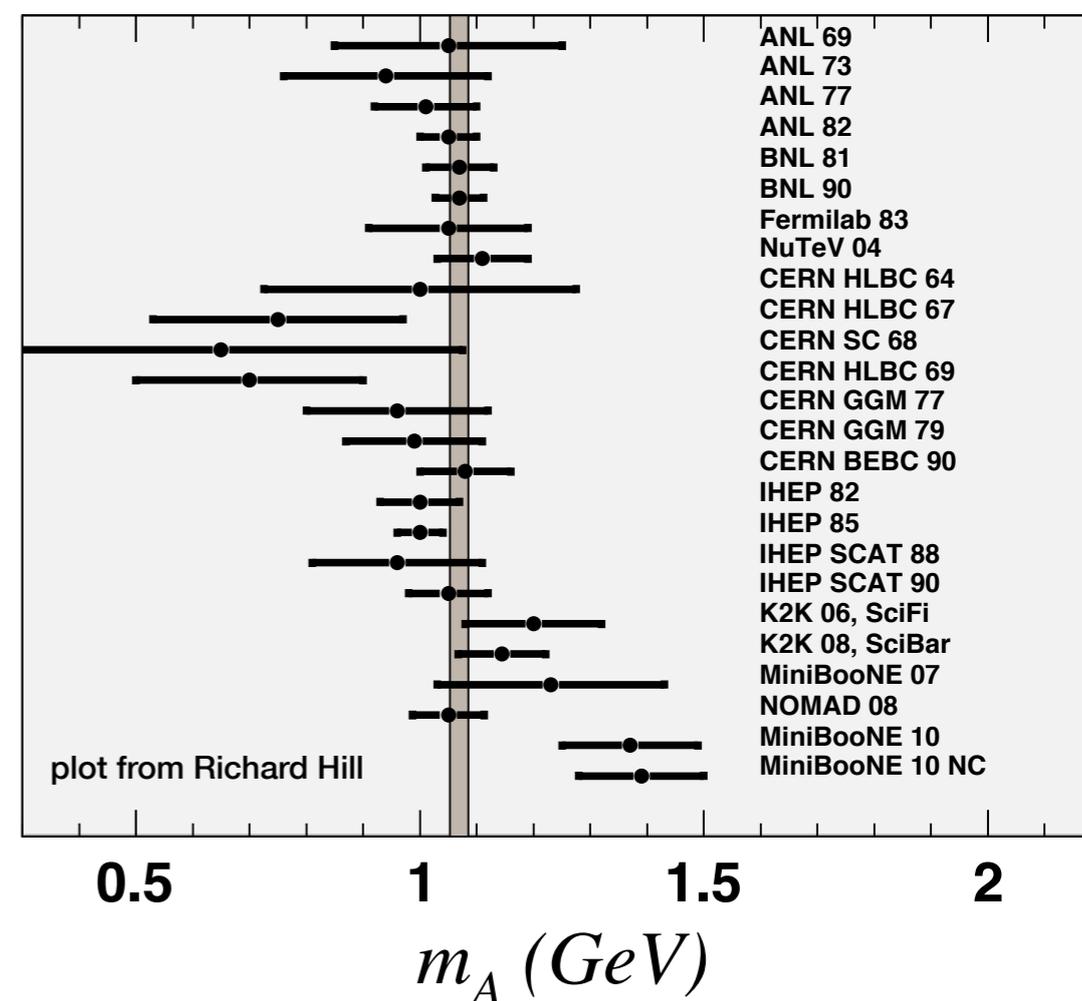
- Cross section for quasielastic  $\nu n \rightarrow e^- p$  and  $\bar{\nu} p \rightarrow e^+ n$  scattering needed to predict signal and background rates in neutrino oscillation searches in few-GeV beams.

*"In this [energy] region, the cross sections even off free nucleons are not very well measured (at the 10 – 40% level) and the data are in frequent conflict with theoretical predictions."*

Snowmass Neutrino WG Report, p. 43

- QE scattering parameterized by vector and axial-vector hadronic form factors.
- Modern experiments have outgrown the dipole Ansatz for  $F_A(q^2) = g_A(1 + q^2/m_A^2)^{-2}$ : no single value of  $m_A$  fits all data.
- Lattice QCD + analyticity can reliably provide shape and normalization of  $F_A(q^2)$  (c.f.  $|V_{ub}|$ ).
- USQCD student interfacing the analyticity formalism with the [GENIE MC](#) generator being developed and used by several neutrino collaborations.

cf. Bhattacharya, Hill, Paz, arXiv:1108.0423 [hep-ph]

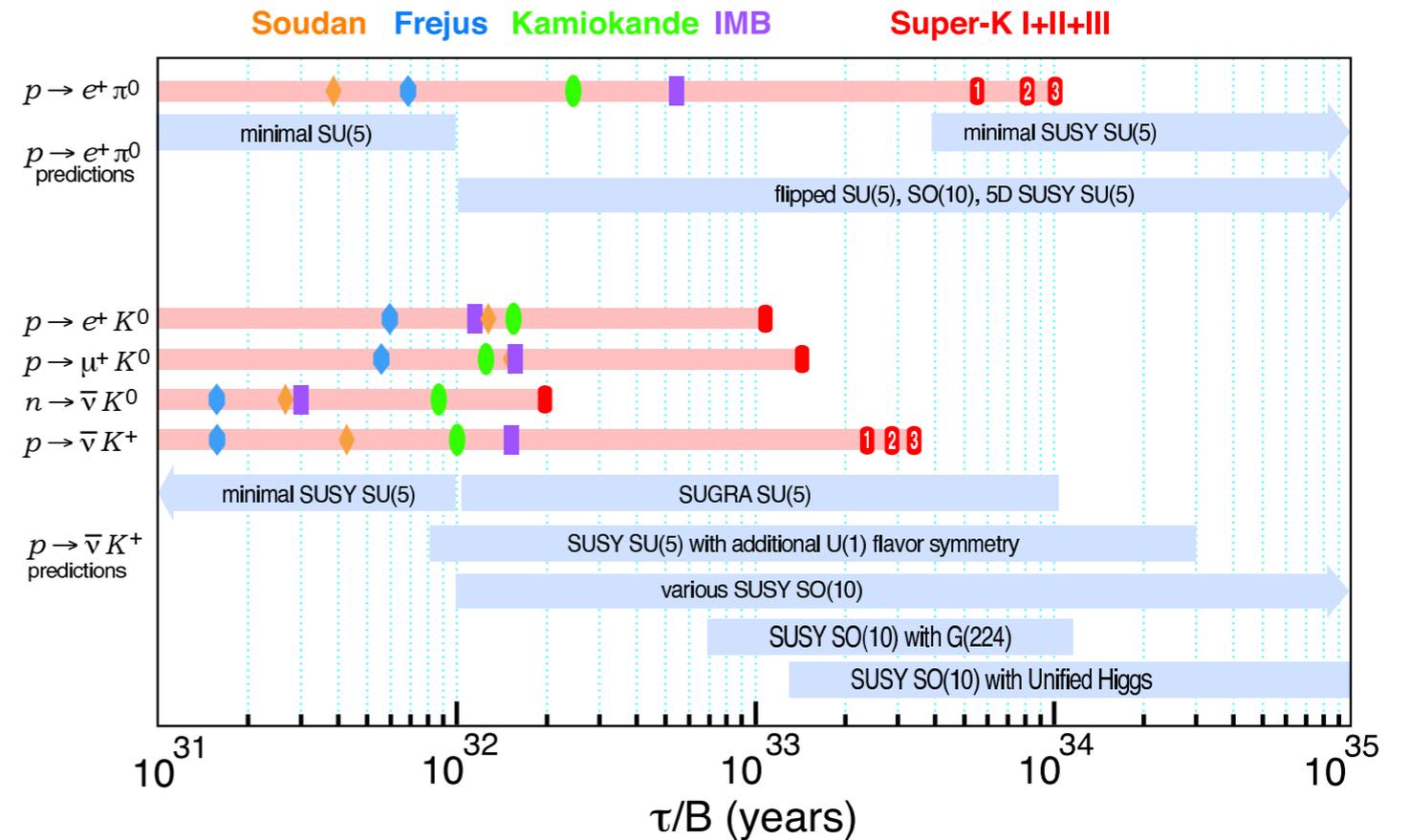


# Proton decay



- Proton decay forbidden in the Standard Model, but a natural prediction of grand unification.
  - ➔ Observation of **proton decay would be unambiguous evidence of new physics.**

- Proton-decay matrix elements essential to interpret limits on (or a measurement of) the proton lifetime as constraints on GUT scenarios.**



- Lattice QCD can compute nonperturbative hadronic matrix elements  $\langle \pi, K, \eta, \dots | \mathcal{O}_{NP} | p \rangle$  of new-physics operators.

- RBC/UKQCD** recently obtained **first three-flavor result for proton-decay matrix elements with a complete systematic error assessment** [Aoki, Shintani, Soni, arXiv: 1304.7424].

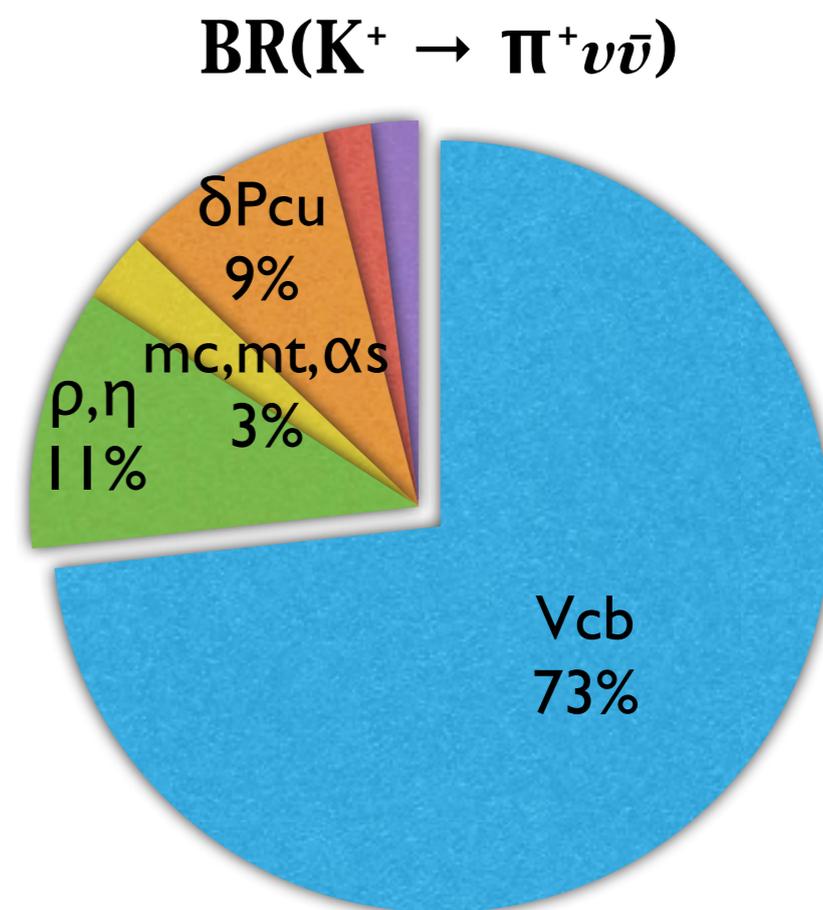
- Errors range from 20-40%, but use of finer lattice spacings and lighter quark masses will improve the precision of these matrix elements in the next five years.



# Quark-flavor physics



- Lattice QCD will continue to be crucial for upcoming kaon and B-physics experiments, e.g.
  - Standard-Model branching ratios for **rare kaon decays**  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  decays limited by  $\sim 10\%$  parametric uncertainty in  $A^4 \propto |V_{cb}|^4$ ;  $|V_{cb}|$  can be obtained from exclusive  $B \rightarrow D^{(*)} l \nu$  decays.
  - Standard-Model predictions for  $B \rightarrow \tau \nu$  and  $B_s \rightarrow \mu^+ \mu^-$  rely on  $f_B$  and  $f_{B_s}$ , respectively.
  - Standard-Model branching fraction predictions for **rare  $b \rightarrow s$  decays** (e.g.  $B \rightarrow K^{(*)} l^+ l^-$ ) limited by hadronic form factor uncertainties.
- Now that field is more mature, **expanding program to more challenging matrix elements such as  $K \rightarrow \pi \pi$  decay** ( $\Delta I = 1/2$  rule and  $\varepsilon'_{K}/\varepsilon_K$ ), **long-distance amplitudes.** (e.g. rare kaon decays,  $D$ -meson mixing), **and multi-hadron final states** (e.g.  $D \rightarrow \pi \pi, K K, \dots$ ).

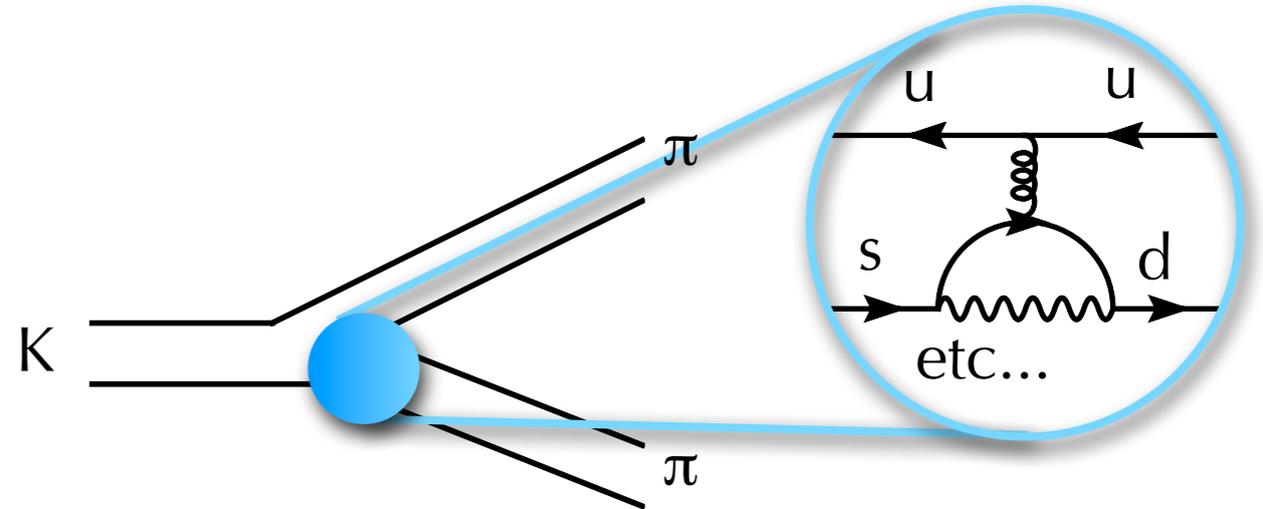


[Brod & Gorbahn  
PRD83 (2011) 034030]

# $K \rightarrow \pi\pi$ decay and $\epsilon'/\epsilon$

- Direct CP-violation in  $K \rightarrow \pi\pi$  decays ( $\epsilon'_K/\epsilon_K$ ) sensitive to new physics because it receives contributions from EW penguins.

- Measured experimentally to <10% precision, but **utility for testing Standard Model handicapped by large uncertainty in corresponding weak matrix elements.**



- In the past two years, the **RBC-UKQCD Collaboration** made significant progress in resolving theoretical issues associated with computing  $K \rightarrow \pi\pi$  amplitudes.
  - Computed  $\Delta I=3/2$  matrix elements with nearly physical pion and kaon masses, and obtained  $\text{Re}(A_2)$  &  $\text{Im}(A_2)$  with  $\sim 20\%$  errors (**PRL 108 (2012) 141601**).
  - Performed pilot study of  $\Delta I=1/2$  matrix elements with  $\sim 330$  MeV pions (**PRD 84 (2011) 114503**).
- **Simulations with physical pions on BG/Q at ALCF should yield first *ab initio* QCD calculation of  $\Delta I=1/2$  rule and  $\epsilon'_K/\epsilon_K$  with  $\sim 20-30\%$  precision in one or two years.**
- With projected lattice improvements, **combining the pattern of results for  $\epsilon'_K/\epsilon_K$  with  $K \rightarrow \pi\nu\nu$  decays can help distinguish between new-physics scenarios** (Buras et al., NPB **566** (2000)).

# Non-Standard Model Higgs Physics



*For “theoretical models of the Higgs field ...big ideas are proposed: supersymmetry, higher dimensions, Higgs compositeness...”*

Peskin Snowmass Energy Frontier Introduction, p 7.

- **Many new-physics scenarios with nonperturbative dynamics** can be studied quantitatively using lattice-field-theory technology.
- **Composite Higgs:** Higgs particle could be a dilaton or a pseudo-Goldstone boson.
  - New lattice methods have been developed to identify conformal or near-conformal behavior in gauge theories beyond QCD by studying the running coupling, mass anomalous dimension, and chiral-symmetry breaking pattern.
  - Current highest priority to find a viable model with a light scalar and **S-parameter** consistent with precision EW constraints.
  - Can then make predictions for the particle **spectrum and modifications to W-W scattering** that can be tested at the 14-TeV **LHC** run or future colliders.
- **Supersymmetry:** SUSY-breaking mechanism could be dynamical.
  - Simple candidate for SUSY-breaking sector is a supersymmetrized version of QCD.
  - **Recent** lattice efforts focused on studying **super-Yang Mills**, but **long-term** goal is to simulate super-QCD and compute the **soft parameters of the low-energy theory**.
  - Will constrain existing SUSY models and provide inputs for SUSY model building.



# Spinoffs: LGT and computing in industry

- **Ken Wilson**, inventor of lattice gauge theory, was an early proponent of supercomputing. (In the 70s, he was programming array processors in assembly language to attack critical phenomena problems for which he won the Nobel Prize.)
- Academic **purpose-built Lattice QCD computers** (Caltech (Cosmic Cube), Columbia, IBM (GF11, not a commercial project), Fermilab ACPMAPS, ...) were influential in establishing the parallel computing model later adopted by industry.
- **The IBM Blue Gene line** was spearheaded by Al Gara, who began in supercomputing at Columbia on Norman Christ's lattice QCD computers. (The Columbia group helped design the Blue Gene/Q, such as Mira at the Argonne Leadership Computing facility.)
- **NVIDIA** (maker of the GPUs in Titan at the Oak Ridge Leadership computing facility) has hired two lattice gauge theorists. They help design future **GPU-based supercomputers** by evaluating how they would run lattice QCD algorithms.



# Spinoffs: LGT and computing in experimental HEP

- In the coming decade, we will see increasing complexity in chips, with increasing hierarchies of processors and memory on each chip.
- **GPUs**: very complex compute modules and memory, special programming language (CUDA) required.
  - USQCD has made its codes run very efficiently, at the cost of many person-years of effort.
  - Fermilab's lattice computing group proselytized for GPUs at the lab and found interest from the Monte Carlo builders.
- The Fermilab lattice QCD computing group is leading an effort to investigate the applicability of **Intel's** latest many-core chip, the **Phi**, with interest from many areas in HEP computing
  - Lattice QCD, off-line track reconstruction, on-line reconstruction and high-level triggers, GEANT parallelization, accelerator modeling, DES data processing, ...
  - **Intel** also seems to be entering the supercomputing market.



# Summary 1

- The need for lattice calculations in the coming decade will be almost ubiquitous in the HEP experimental.
  - In **charged lepton physics**:  $g-2$  (vacuum polarization and light by light scattering),  $\mu 2e$ ,  $\mu e \gamma$  (nucleon scalar matrix elements).
  - In **neutrino physics**: LBNE, Nova, ... (nucleon axial M. E.).
  - In **dark matter experiments**: CDMS, ... (nucleon scalar matrix elements).
  - In **underground proton decay experiments**: LBNE, ... (proton decay matrix elements).
  - In **quark flavor physics**: Belle-2, LHCb, ... (many matrix elements with much better precision).
  - At the **LHC**: CMS, Atlas (strongly coupled BSM sectors).
  - At the **ILC, TLEP, muon colliders**: (very high precision  $\alpha_s$  and  $m_b$ ).

# Summary 2

- USQCD dominated quark-flavor physics on the lattice during the era of the flavor factories.
- In the coming decade, new lattice calculations will be needed throughout the HEP experimental program.
- We have systematically examined the future US experimental program and talked with experimenters to determine the lattice calculations that need to be done.
- The LQCD-ext II project will help us complete them.
  - Continued support of the U.S. lattice-QCD effort is essential to fully capitalize on the enormous investment in the HEP experimental program.
  - We hope that you agree.



# Extra slides



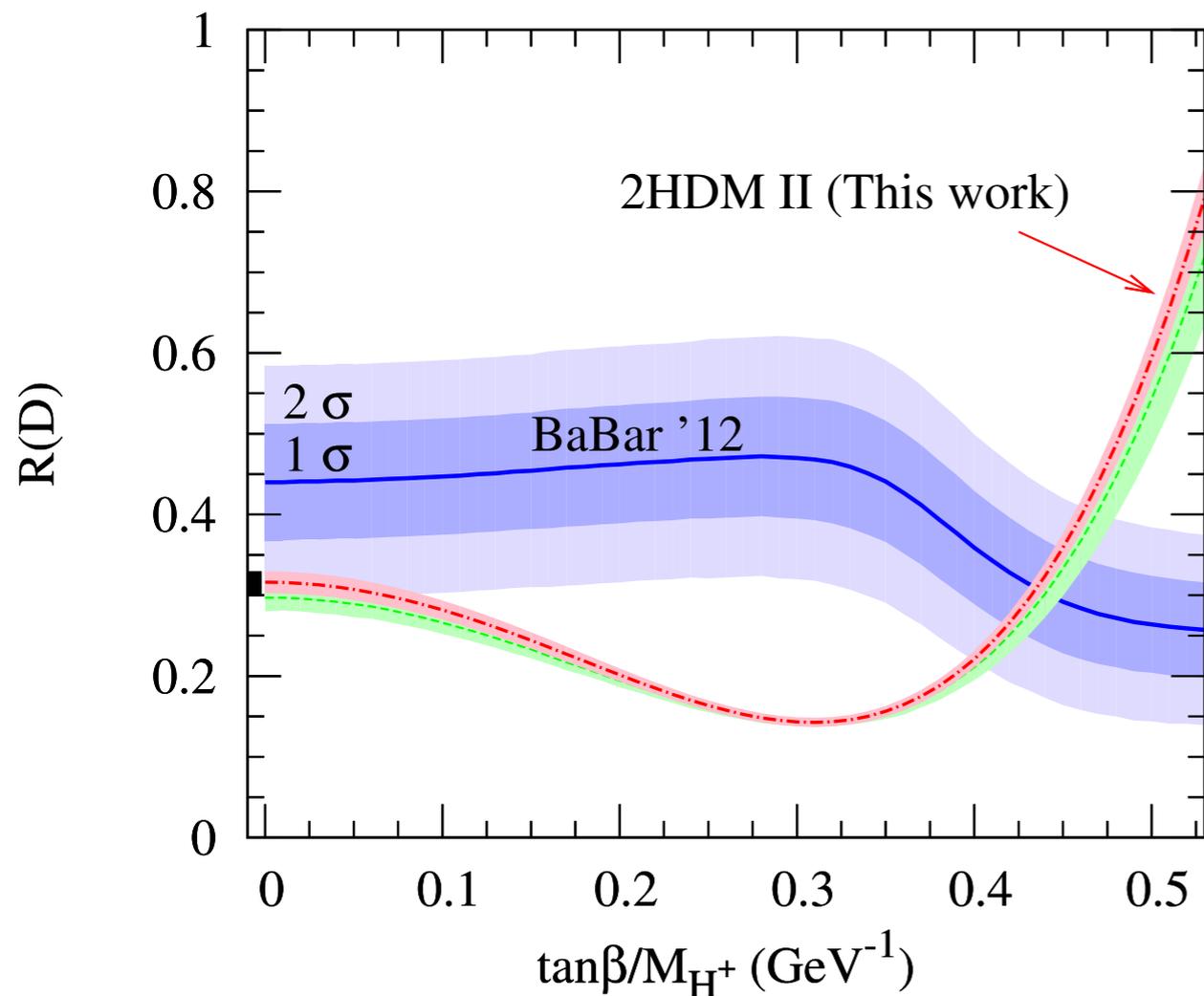
# Scientific motivation

- The experimental **HIGH-ENERGY PHYSICS** community is presently searching for new physics with two complementary approaches
  - (1) **The Energy Frontier:** directly produce new particles at high-energy colliders
  - (2) **The Intensity Frontier:** observe deviations from the Standard Model in precise measurements of rare processes that probe quantum-mechanical loop effects
- **Comparison between measurements and Standard-Model predictions often limited by nonperturbative matrix elements**
- The experimental **NUCLEAR PHYSICS** community aims to
  - (1) Determine the structure, spectrum, and interactions of hadrons
  - (2) Understand the properties of matter under extreme conditions
- **Lattice QCD enables the determination of hadron properties from first principles**
- **Lattice gauge theory is crucial to maximize the scientific output of the current and future high-energy and nuclear physics experimental programs**



# 2012 Highlight: R(D) from unquenched lattice QCD

- ◆  $B \rightarrow D^{(*)} \tau \nu$  decays sensitive to new-physics contributions such as from charged Higgs bosons
- ◆ Recently **BaBar** measured the ratios  $R(D) = \text{BR}(B \rightarrow D \tau \nu) / \text{BR}(B \rightarrow D l \nu)$ ,  $R(D^*) = \text{BR}(B \rightarrow D^* \tau \nu) / \text{BR}(B \rightarrow D^* l \nu)$  and observed excesses in both channels that **disagree with the Standard Model by  $3.4\sigma$**  [[PRL 109 \(2012\) 101802](#)]



- ◆ **Fermilab Lattice and MILC Collaborations** quickly followed with **first Standard-Model calculation of R(D) from *ab initio* lattice-QCD** [[PRL 109 \(2012\) 071802](#)]
- ◆ Uncertainty smaller than previous model estimate from dispersive bounds, heavy-quark symmetry, and quenched lattice QCD
- ◆ Lattice calculation of  $R(D^*)$  *in progress*...

# 2012 Highlight: $K \rightarrow \pi \ell \nu$ form factor in the continuum

- ◆  $K \rightarrow \pi \ell \nu$  form factor can be combined with experimentally-measured branching fraction to obtain  $|V_{us}|$  in the Standard Model:

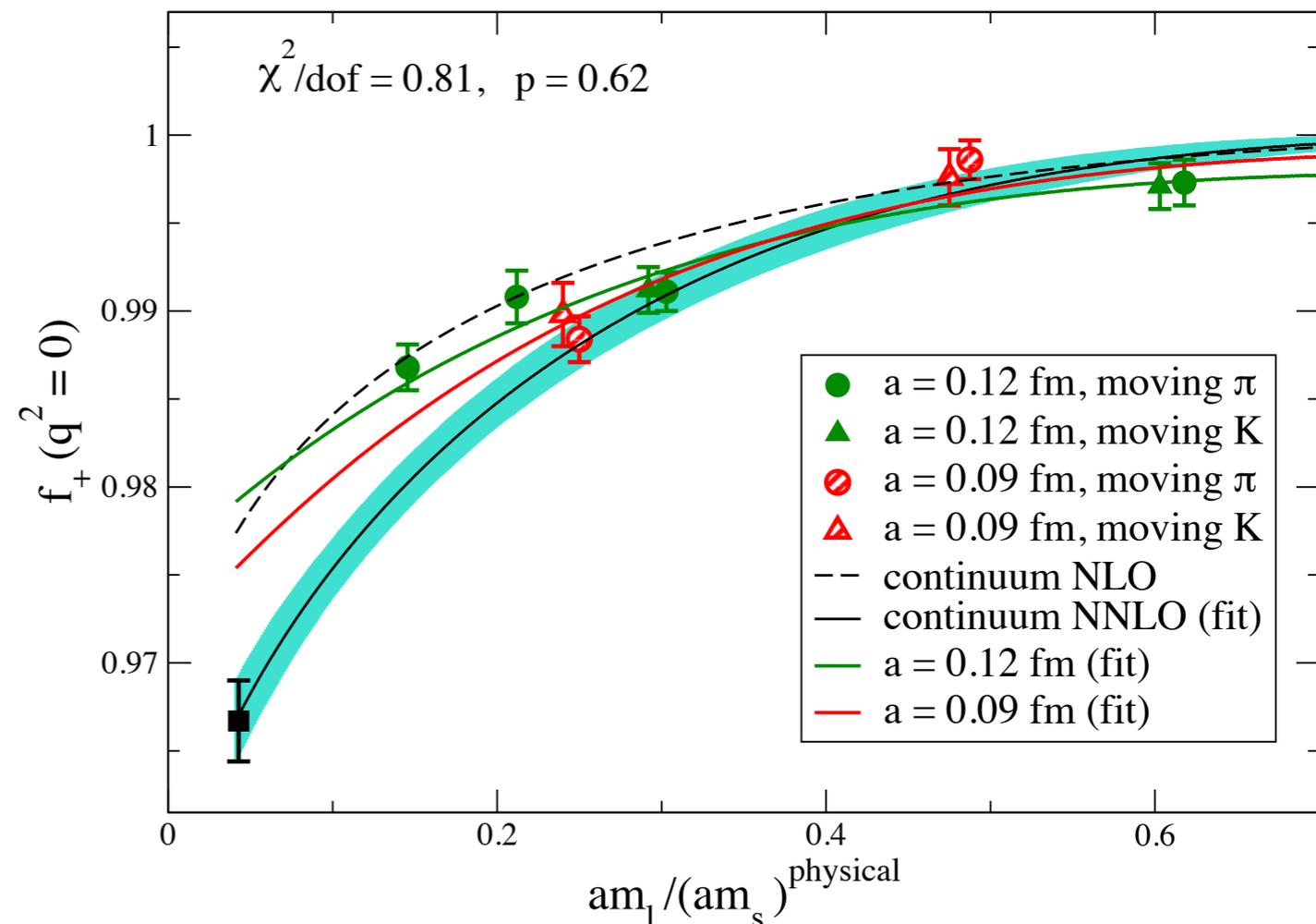
$$\Gamma(K \rightarrow \pi \ell \nu) = \frac{G_F^2 m_K^5}{192 \pi^3} C_K^2 S_{EW} |V_{us}|^2 |f_+^{K^0 \pi^-}(0)|^2 I_{K\ell} \left(1 + \delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right)^2$$

- ◆ **Fermilab Lattice and MILC** recently obtained the **first three-flavor result for the  $K \rightarrow \pi \ell \nu$  form factor at zero momentum transfer with two lattice spacings** and a controlled continuum extrapolation [**Bazavov et al. PRD87, 073012 (2013)**]

- ◆ Single most precise result for  $f_+(0)$  enables 0.5% determination of  $|V_{us}|$

$$f_+^{K\pi}(0) = 0.9667(23)_{\text{stat}}(33)_{\text{sys}}$$

$$|V_{us}| = 0.2238(9)_{\text{theo}}(5)_{\text{exp}}$$



# 2013 Highlight: $f_K/f_\pi$ at the physical point

- ◆ The SU(3) flavor-breaking ratio  $f_K/f_\pi$  allows a determination of  $|V_{ud}|/|V_{us}|$  [Marciano]

$$\frac{\Gamma(K \rightarrow l\bar{\nu}_l)}{\Gamma(\pi \rightarrow l\bar{\nu}_l)} = \left(\frac{|V_{us}|}{|V_{ud}|}\right)^2 \left(\frac{f_K}{f_\pi}\right)^2 \frac{m_K \left(1 - \frac{m_l^2}{m_K^2}\right)^2}{m_\pi \left(1 - \frac{m_l^2}{m_\pi^2}\right)^2} \left[1 + \frac{\alpha}{\pi}(C_K - C_\pi)\right]$$

- ◆ **MILC collaboration** recently obtained the **first lattice-QCD determination of  $f_K/f_\pi$** 
  - (1) including dynamical charm and
  - (2) at the physical pion mass with highly-improved staggered (HISQ) quarks [Bazavov *et al.* PRL110, 172003]

Source	$f_{K^+}/f_{\pi^+}$
Monte-Carlo statistics	0.22%
Continuum extrapolation	0.28%
Finite-volume corrections	0.14%
EM corrections	0.02%
Total	0.38%

- ❖ **Eliminate error from extrapolation to physical u- and d-quark masses**

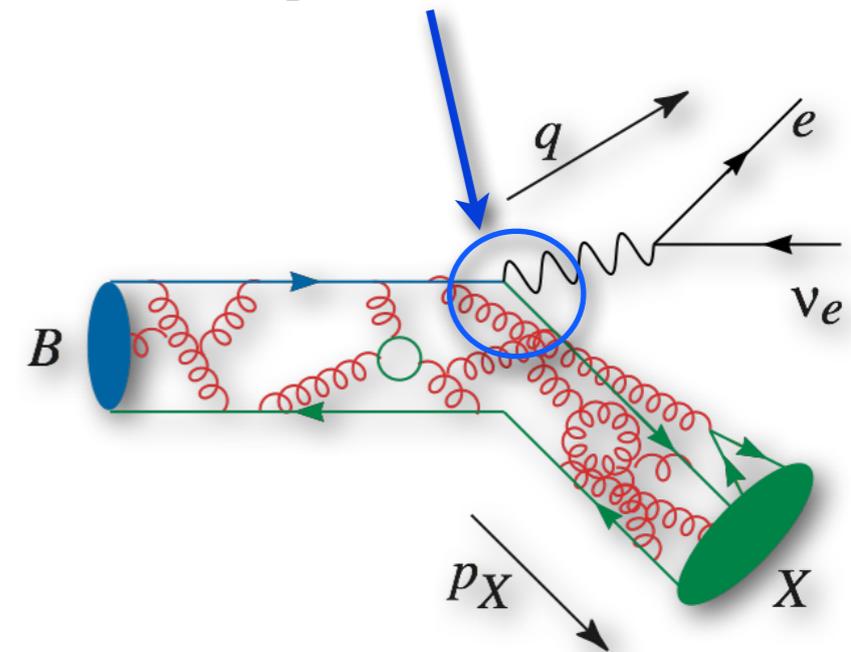
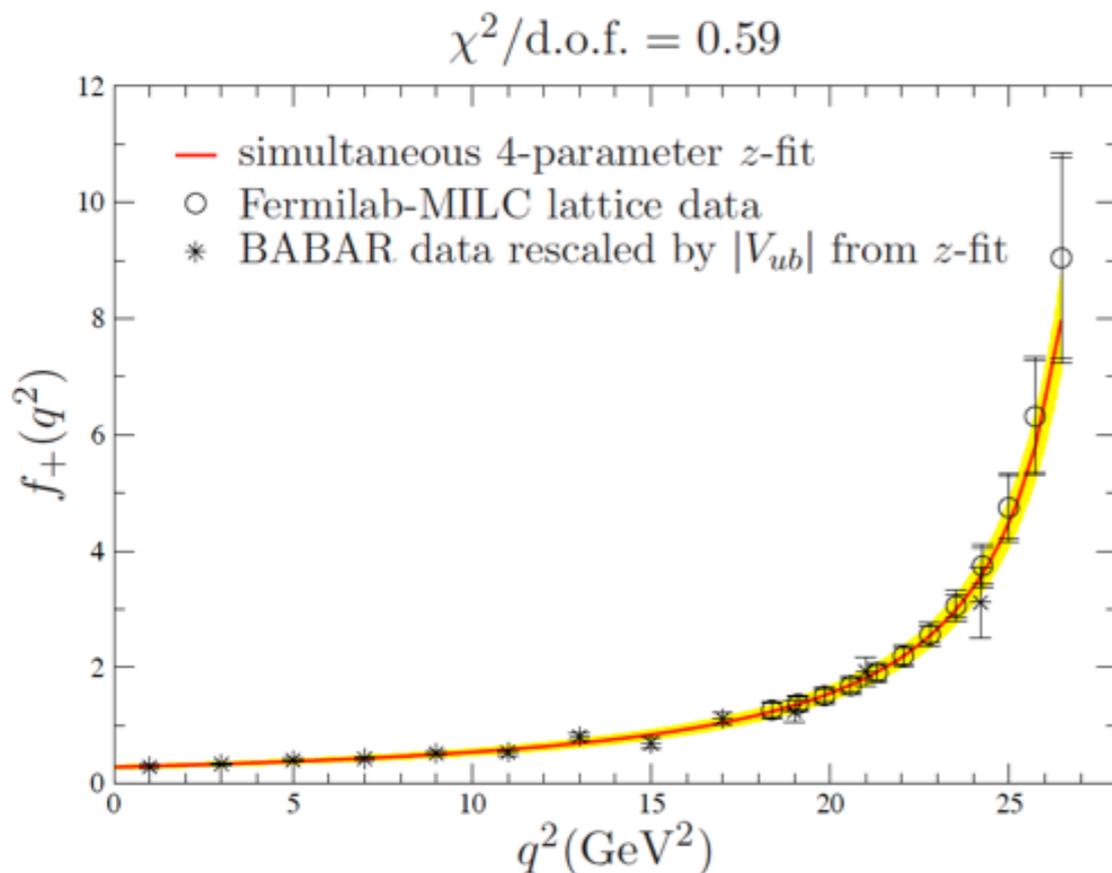
- ◆ Combined with  $|V_{ud}|$  from nuclear  $\beta$ -decay, enables sub-percent test of unitarity of 1<sup>st</sup> row of CKM matrix

$$1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = 0.0003(6)$$

# Determination of $V_{ub}$ from $B \rightarrow \pi \ell \nu$

The CKM matrix element  $V_{ub}$  can be determined from the observed rate for the decay of a B meson into a pion and two leptons,  $B \rightarrow \pi \ell \nu$ , and the lattice QCD calculation of the form factor parameterizing the strong interaction effects.

$$\frac{d\Gamma(B^0 \rightarrow \pi^- \ell^+ \nu)}{dq^2} = \frac{G_F^2}{192\pi^3 m_B^3} \left[ (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2 m_\pi^2 \right]^{3/2} |V_{ub}|^2 |f_+(q^2)|^2$$



Few percent determination of exclusive  $|V_{ub}|$  challenging:  
 both lattice and experimental results are momentum dependent and highly correlated.

# $B \rightarrow \pi/\nu$

Phys. Rev. **D79**:054507, 2009.

Rewrite form factors

$$f(t) = \frac{1}{P(t)\phi(t, t_0)} \sum_{k=0}^{\infty} a_k(t_0) z(t, t_0)^k$$

$$z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$-.3 < z < .3$$

Unitarity + analyticity:  
 $\sum a_k^2 \approx 1$ .

Heavy quark theory  
 (Becher, R. Hill):  
 $\sum a_k^2 \sim (\Lambda_{\text{QCD}}/m_b)^3$ .

PHYSICAL REVIEW D **83**, 071101(R) (2011)

Measurement of the decay  $B^0 \rightarrow \pi^- \ell^+ \nu$  and determination of  $|V_{ub}|$

(The Belle Collaboration)

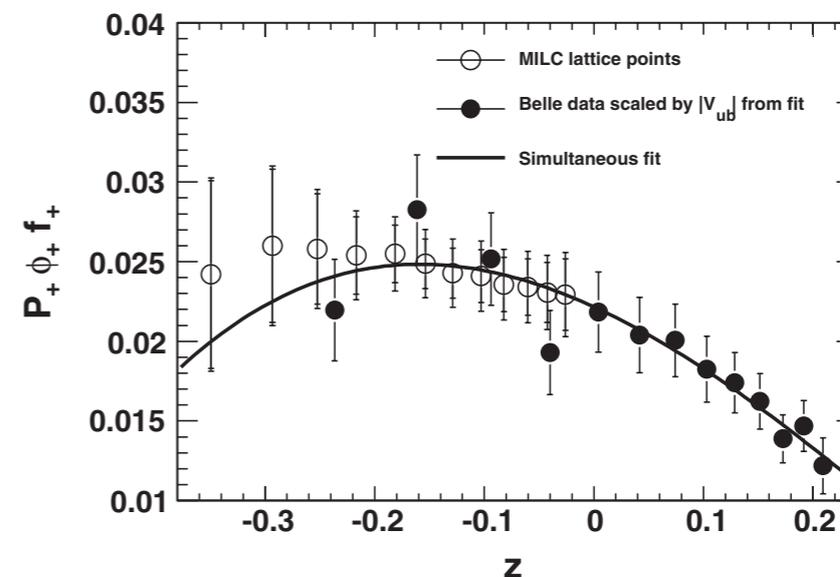


FIG. 3.  $|V_{ub}|$  extraction from a simultaneous fit of experimental (closed circles) and FNAL/MILC lattice QCD results (open circles) [9]. The error for each experimental data point is the total experimental uncertainty. The smaller error bars of the lattice QCD results are statistical only while the larger ones also include systematic uncertainties.

$$|V_{ub}| = (3.43 \pm 0.33) \times 10^{-3}$$

Recent BaBar result also uses our work.

Most accurate calculation  
 of exclusive determination  
 of  $V_{ub}$  so far.

# $B \rightarrow \pi/\nu$

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Rewrite form factors

$$f(t) = \frac{1}{P(t)\phi(t, t_0)} \sum_{k=0}^{\infty} a_k(t_0) z(t, t_0)^k$$

Accounts for  $B^*$  pole  
(like Becirevic-Kaidalov  
parameterization).

PT calculable  
function to make  
 $a_k$ s look simple.

$$z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

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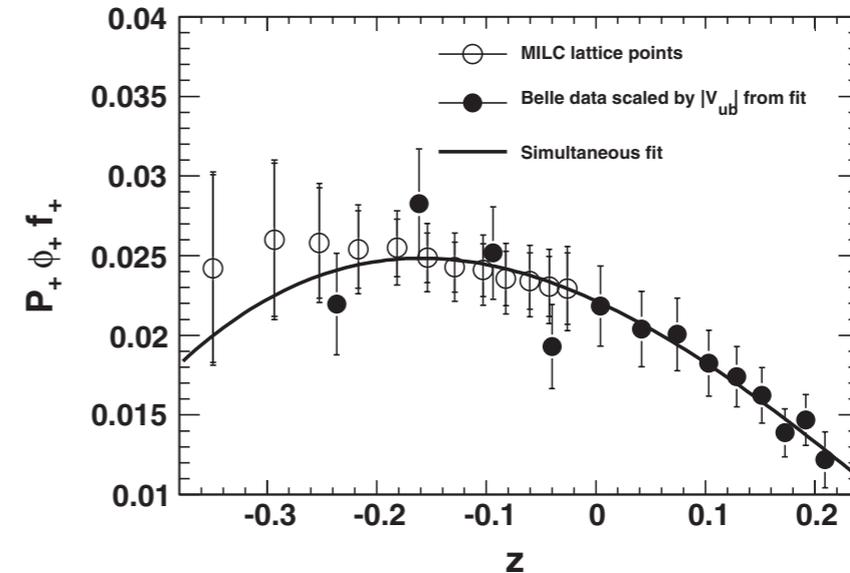


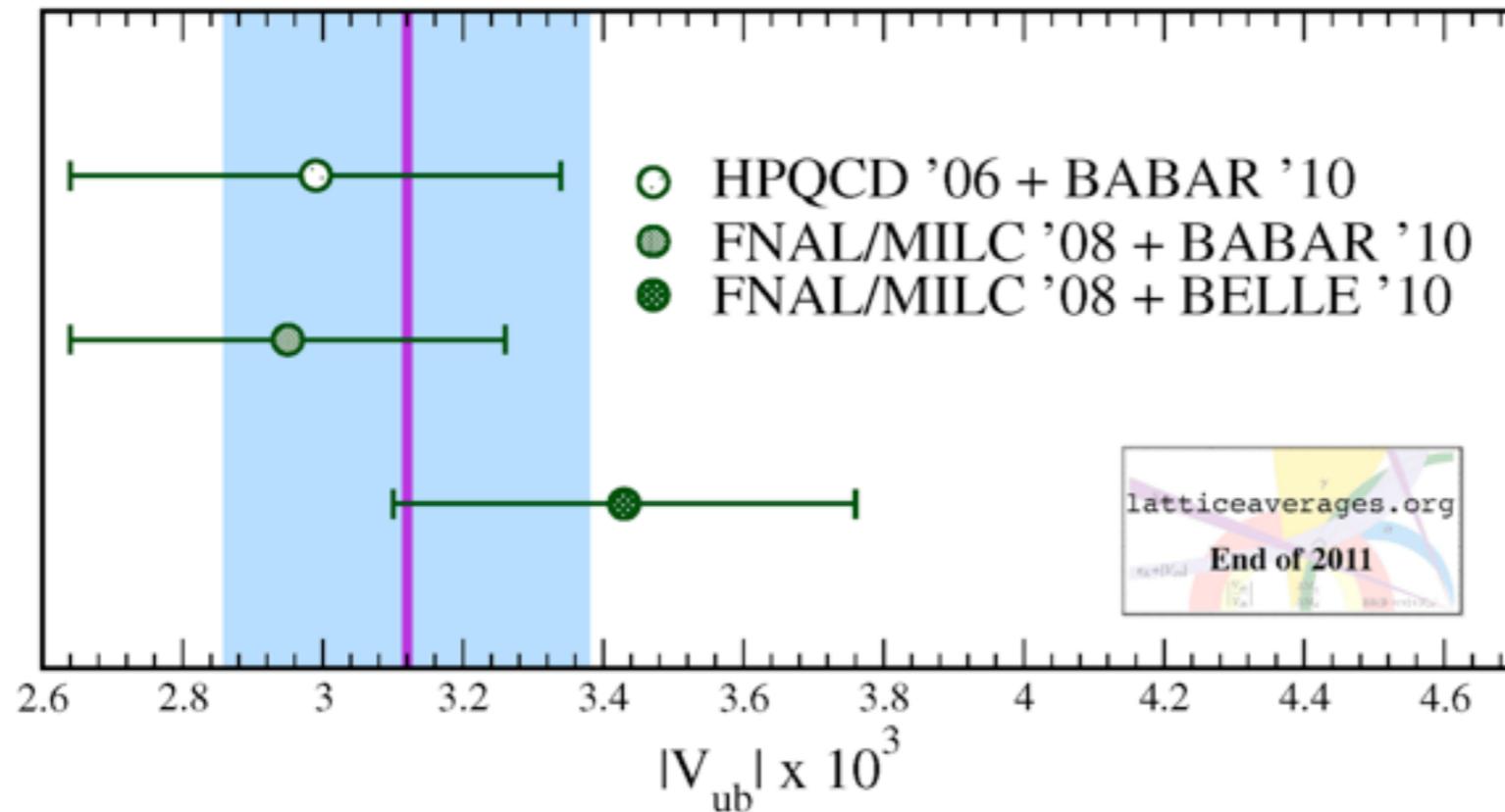
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# Determination of $V_{ub}$ from $B \rightarrow \pi l \nu$



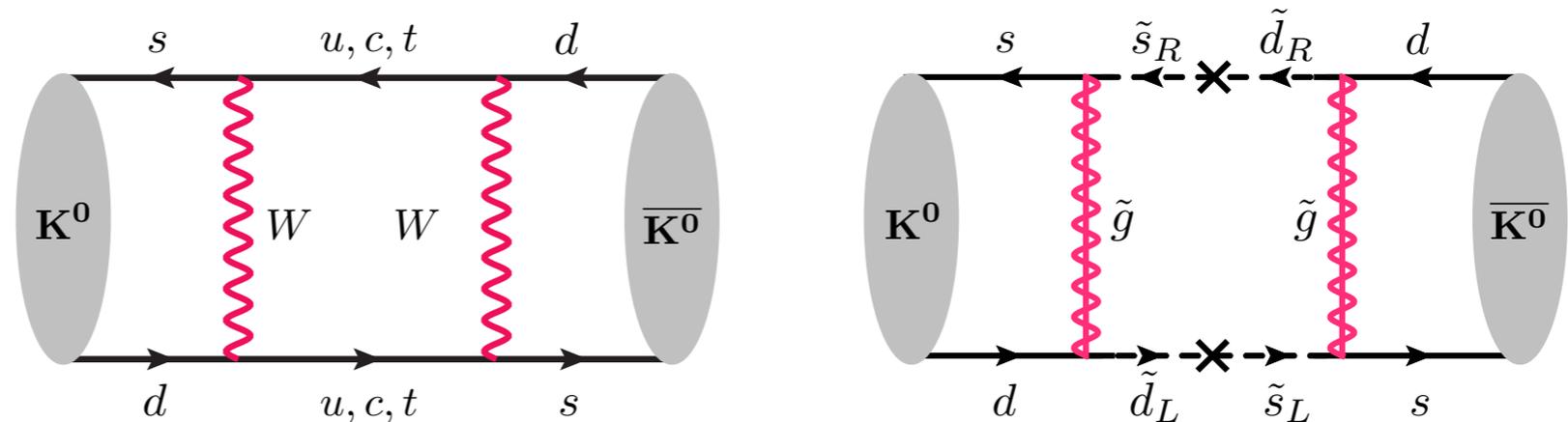
Uncertainty from combined fit  $\sim 8\%$ .

- Several groups have improved calculations underway;
- Should be able to reduce the lattice errors in  $|V_{ub}|$  to the current experimental level of  $\sim 4\text{-}6\%$

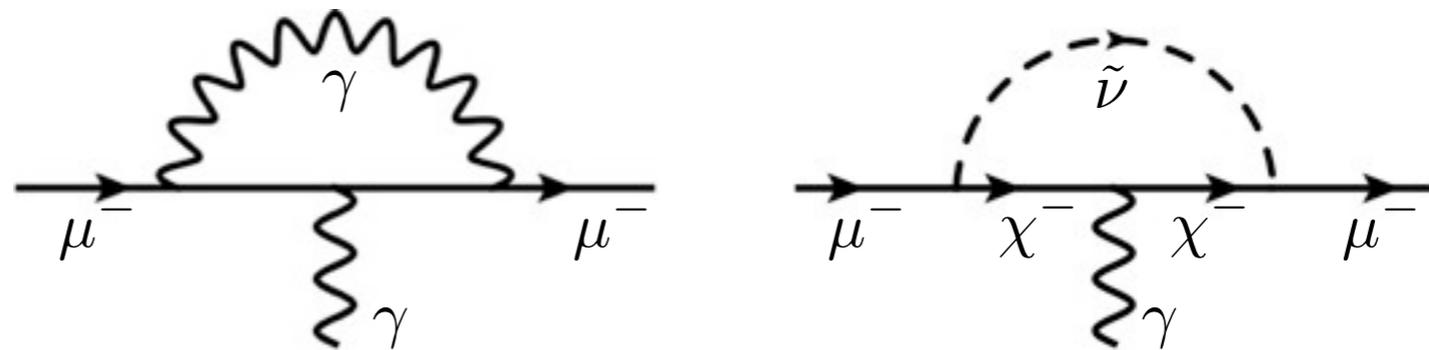
# Why search at the intensity frontier?

- ◆ Precision measurements probe quantum-mechanical loop effects, e.g.:

neutral kaon mixing

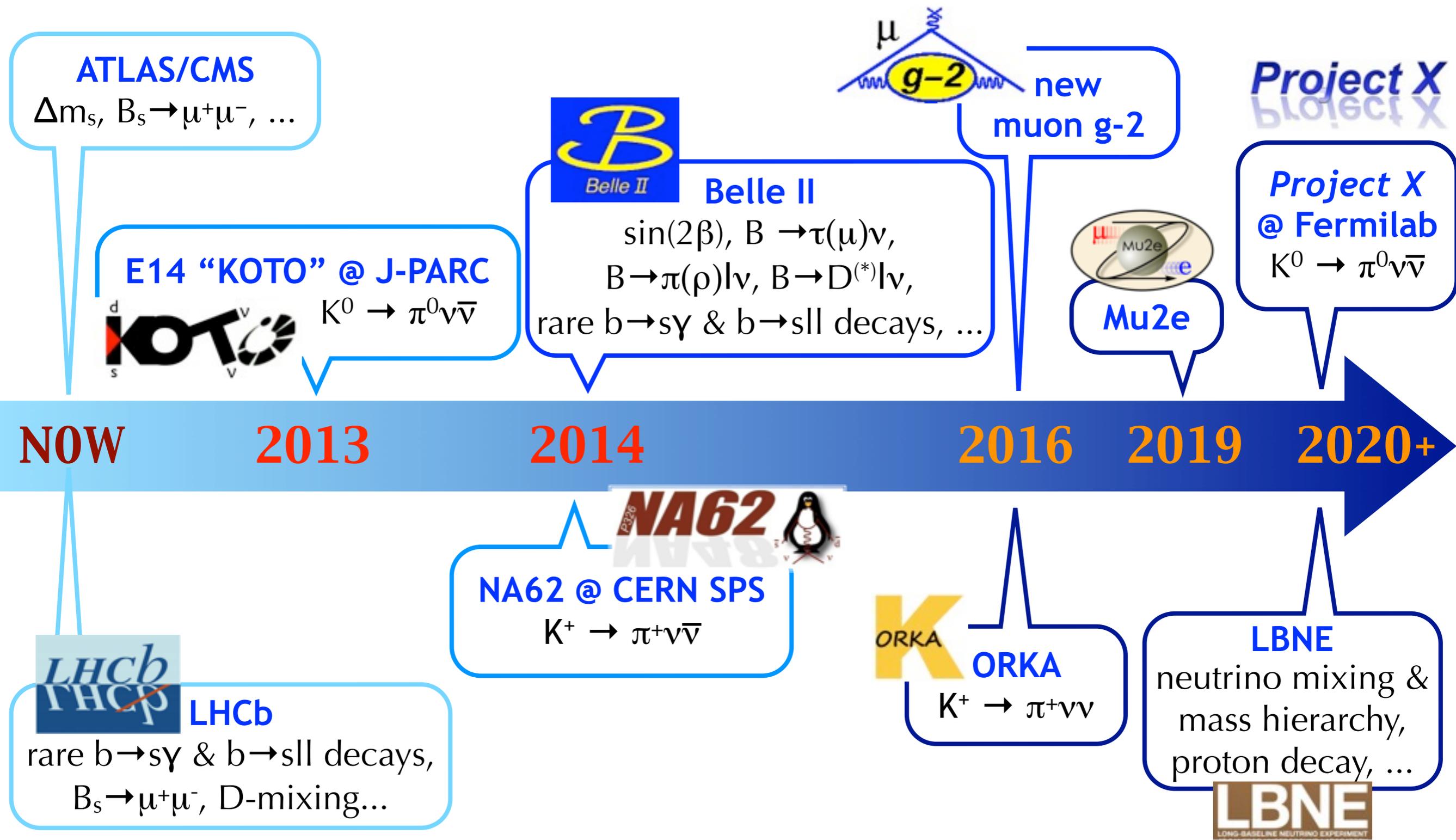


muon g-2



- ◆ Sensitive to physics at higher energy scales than those probed at LHC, in some cases  $O(1,000 - 10,000 \text{ TeV})$  [Isidori, Nir, Perez, *Ann.Rev.Nucl.Part.Sci.* 60 (2010) 355]
- ◆ If new particles are discovered at ATLAS & CMS, precise measurements will still be needed to extract the flavor & CP-violating couplings and determine the underlying structure of the theory

# (*Select*) Upcoming experiments



# Scientific goals and 5-year plan

- ◆ **USQCD aims to support the US HEP experimental intensity-physics program** by “improv[ing] the accuracy of QCD calculations to the point where they no longer limit what can be learned from high precision experiments that seek to test the Standard Model” — *USQCD HEP SciDAC-3 proposal*
- ◆ **2013 White Paper “Lattice QCD at the Intensity Frontier”** outlines a program of calculations matched to experimental priorities
  - (1) “Improve the calculation of the matrix elements needed for the CKM unitarity fit”
  - (2) “Calculate ... new, more computationally demanding, matrix elements that are needed for the interpretation of planned (and in some cases old) experiments”
- ◆ Target quantities and precision goals **developed with input from experimentalists and phenomenologists** (primarily quark-flavor and muon  $g-2$ )
  - ◆ Currently in discussions with neutrino and charged-lepton communities to expand lattice-QCD portfolio relevant for LBNE, Mu2e, etc...

# Computational developments

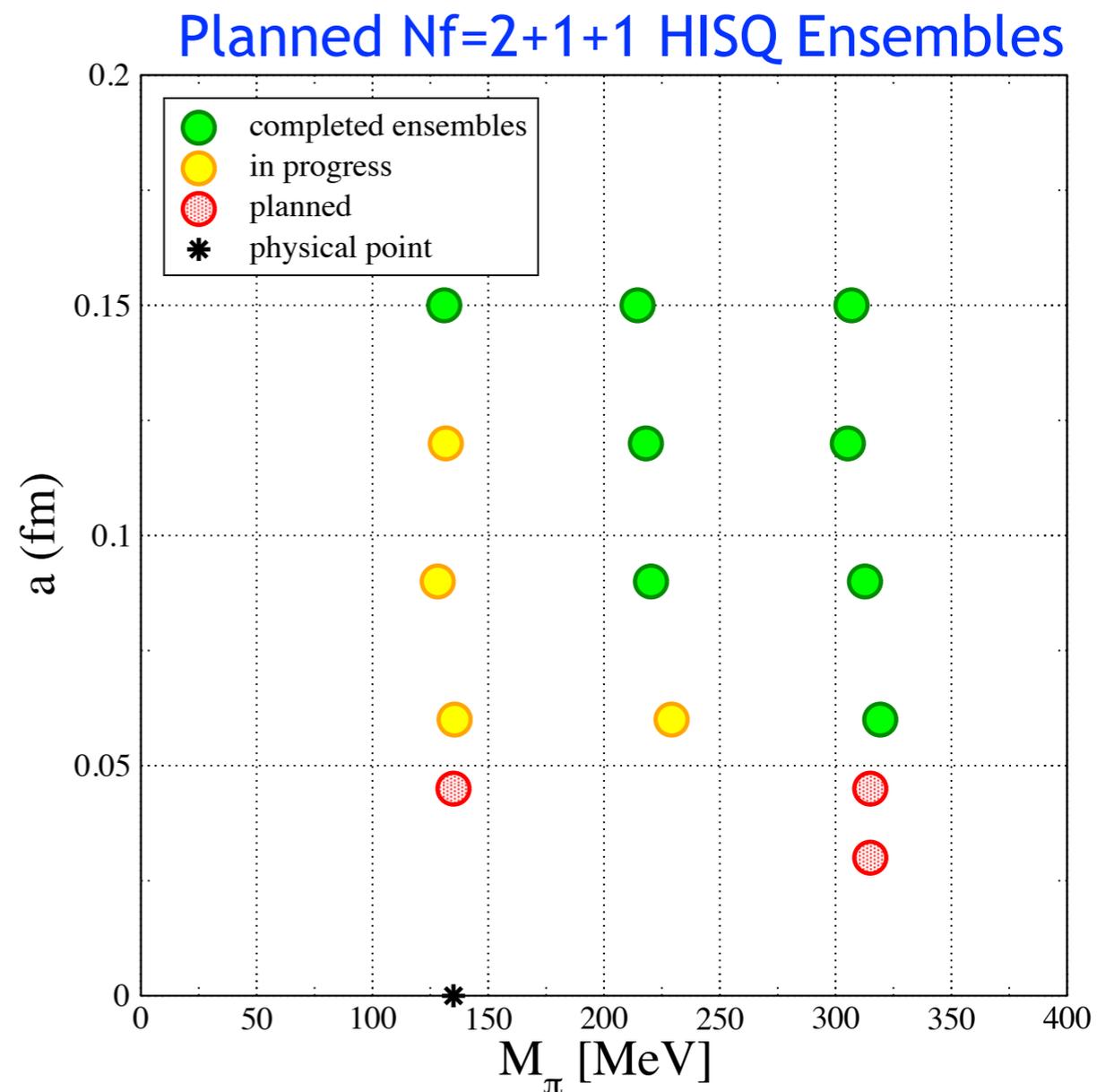
- ◆ **Petascale computing resources** will enable simulations with increased statistics, lighter pions, finer lattice spacings, and larger volumes, thereby helping most sources of uncertainty
- ◆ The following improvements will become widespread over the next five years

## (1) Simulations with physical-mass pions

(2) Systematic inclusion of isospin-breaking and EM

(3) Dynamical charm quarks

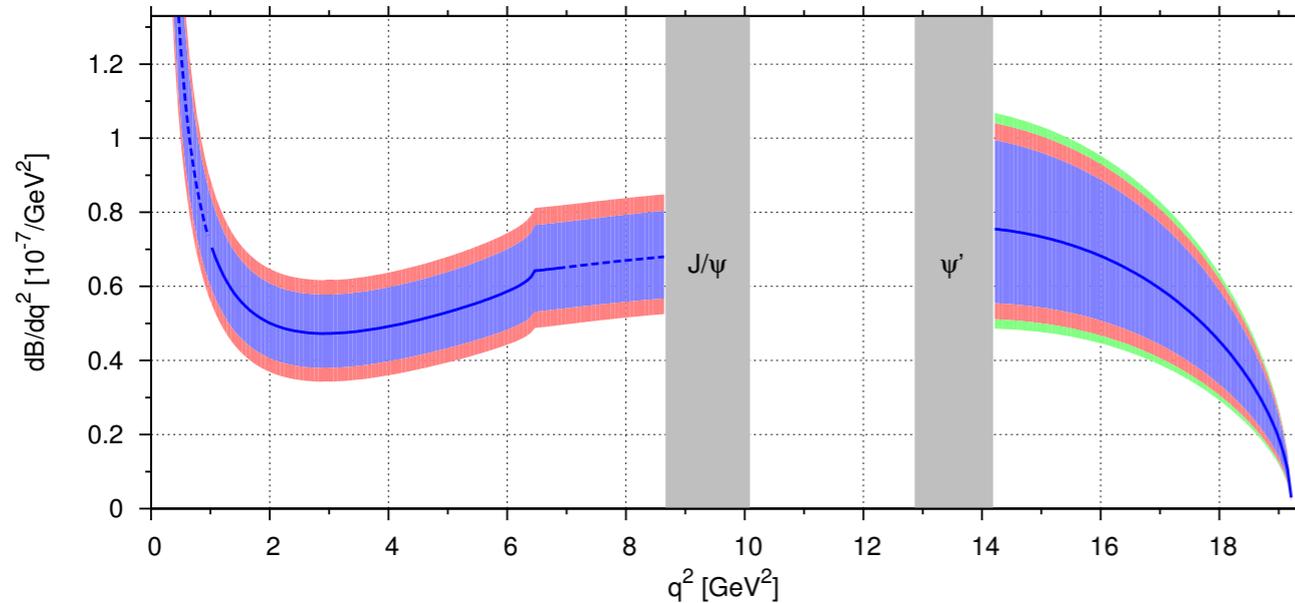
- ◆ **Improved algorithms and analysis methods also being pursued**, but difficult to predict



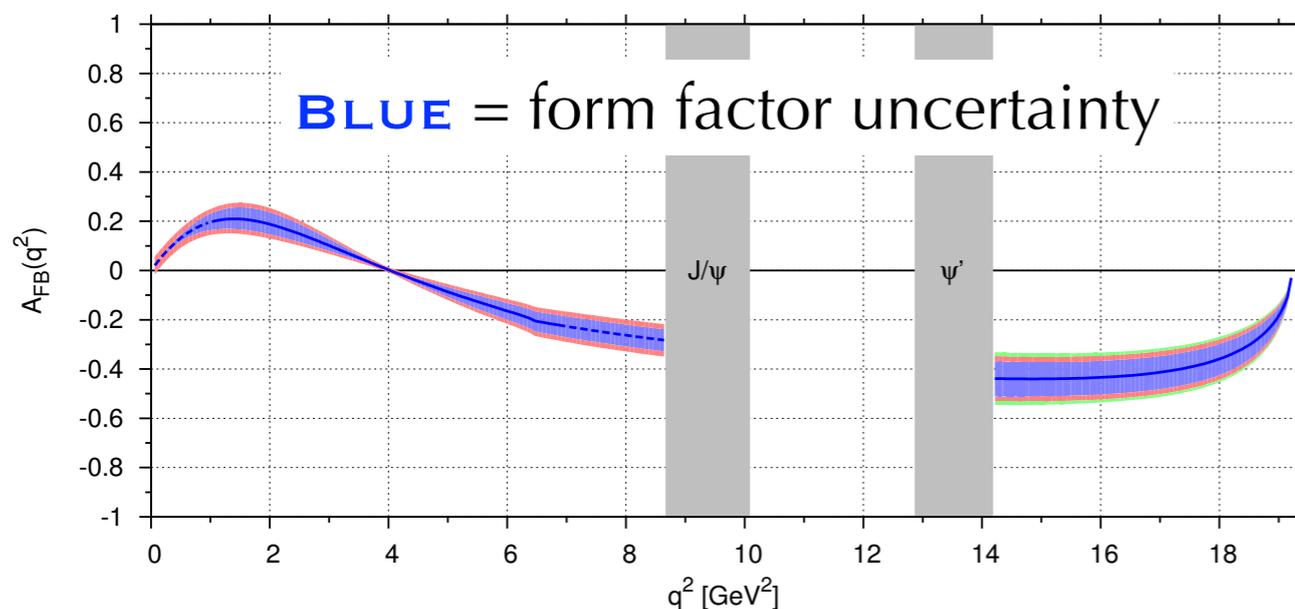
# Rare B decays



## DIFFERENTIAL BRANCHING FRACTION



## FORWARD-BACKWARD ASYMMETRY



◆ Standard Model branching fraction predictions for many  $b \rightarrow s$  processes limited by hadronic form factor uncertainties

◆ E.g., hadronic uncertainties are dominant error for  $B \rightarrow K^* l^+ l^-$  decay observables over all  $q^2$

◆ Lattice QCD calculations are underway of  $B \rightarrow K l^+ l^-$

[Zhou *et al.* (Fermilab/MILC), arXiv: 1111.0981]

and of  $B \rightarrow K^* l^+ l^-$  &  $B \rightarrow K^* \gamma$

[Liu *et al.*, arXiv:1101.2726]

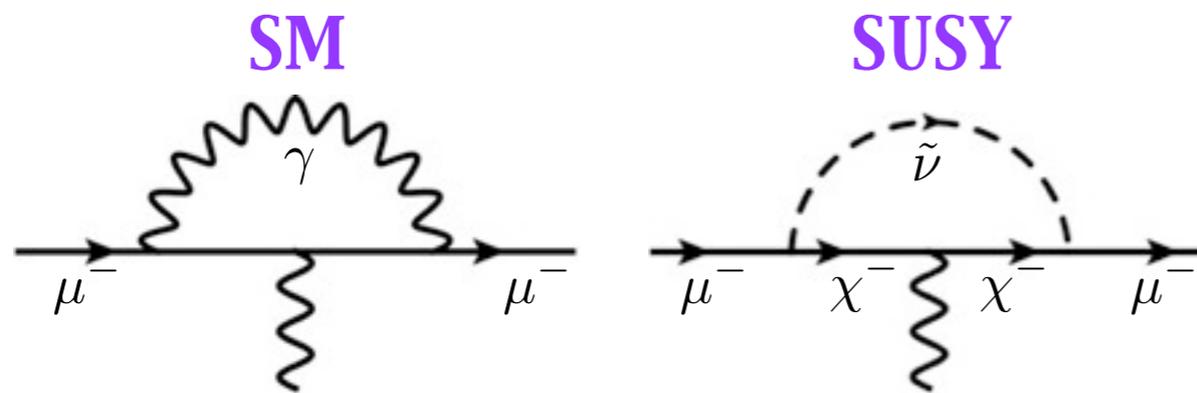
◆ Expect first lattice-QCD results for  $B \rightarrow K l^+ l^-$  by the end of the year with few-percent errors in the high- $q^2$  region

[Bobeth, Hiller, & van Dyk, JHEP 1007 (2010) 098]

# Muon anomalous magnetic moment



- ◆ Sensitive probe of heavy mass scales in the several hundred GeV range
- ◆ New particles can generate significant non-Standard Model contributions in well-motivated models such as Supersymmetry or warped extra dimensions



$$a_{\mu}^{\text{SUSY}} \propto \left( \frac{m_{\mu}}{M_{\text{SUSY}}} \right)^2 \tan(\beta) \text{sign}(\mu)$$

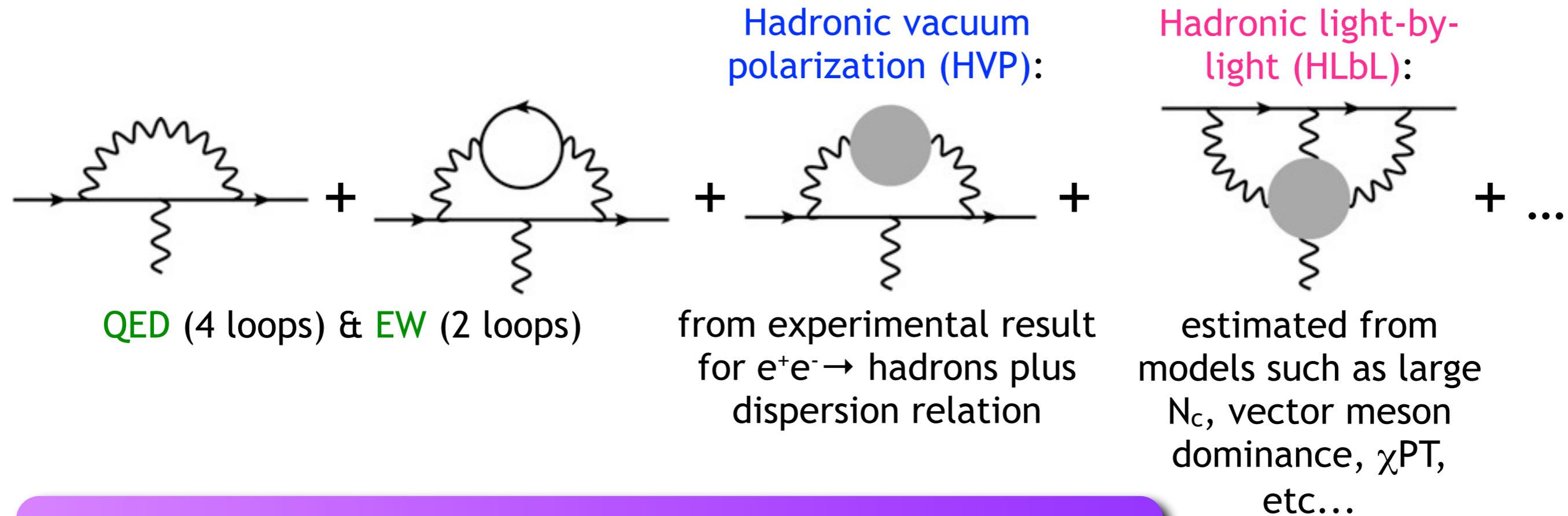
- ◆ Muon g-2 currently measured experimentally to 0.54 ppm

$$a_{\mu}^{\text{exp}} = 116\,592\,089(54)(33) \times 10^{-11} \text{ [E821]}$$

$$a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = 287(80) \times 10^{-11} \text{ [3.6}\sigma\text{]}$$

- ❖ A  $>3\sigma$  discrepancy with the Standard Model, assuming you trust the SM prediction...
- ◆ **New Muon g-2 experiment** will reduce error by a factor of four to 0.14 ppm
  - ❖ Reduction in theoretical uncertainty on SM prediction to the same level (with more reliable error estimate!) needed to conclusively establish the presence of new physics with the planned measurement

# Standard-Model contributions to $g-2$



Contribution	Result ( $\times 10^{11}$ )	Error
QED (leptons)	116 584 718 $\pm$ 0.14 $\pm$ 0.04 $_{\alpha}$	0.00 ppm
HVP(lo) [1]	6 923 $\pm$ 42	0.36 ppm
HVP(ho)	-98 $\pm$ 0.9 $_{\text{exp}}$ $\pm$ 0.3 $_{\text{rad}}$	0.01 ppm
HLbL [2]	105 $\pm$ 26	0.22 ppm
EW	154 $\pm$ 2 $\pm$ 1	0.02 ppm
Total SM	116 591 802 $\pm$ 49	0.42 ppm

[1] Davier, Hoecker, Malaescu, Zhang, Eur.Phys.J. C71 (2011) 1515

[2] Prades, de Rafael, Vainshtein, arXiv:0901.030

# Recent lattice-QCD progress

- ◆ Lattice QCD can provide calculations of the hadronic contributions from QCD first principles with controlled uncertainties that are systematically improvable
- ◆ USQCD research and development efforts on both hadronic contributions are ongoing

## HADRONIC VACUUM POLARIZATION

- ❖ Aubin et al., [Phys.Rev. D86 (2012) 054509] introduced model-independent fitting approach based on analytic structure of  $\Pi(Q^2)$  using Padé approximants to eliminate potentially hidden systematic associated with vector-meson dominance fits
- ❖ Theoretical improvements + increased computing resources should enable a lattice-QCD determination with few-percent error on the timescale of New  $g-2$

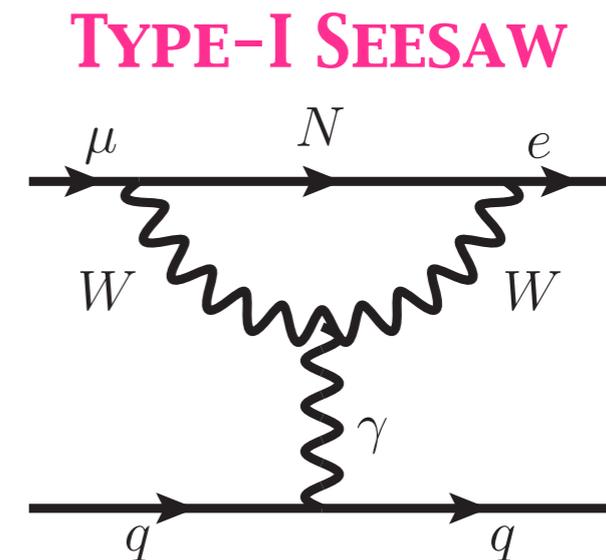
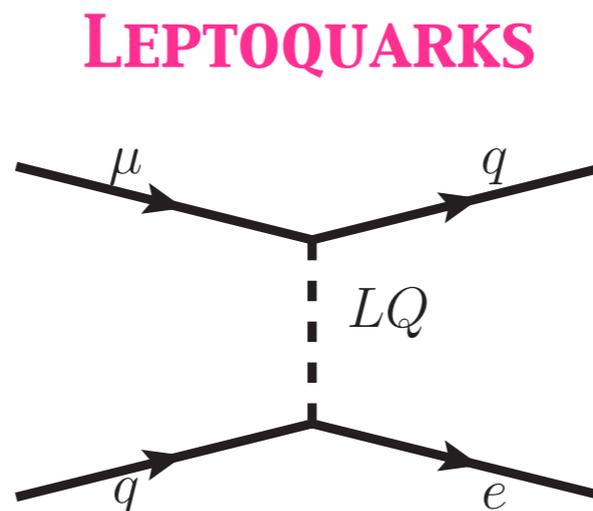
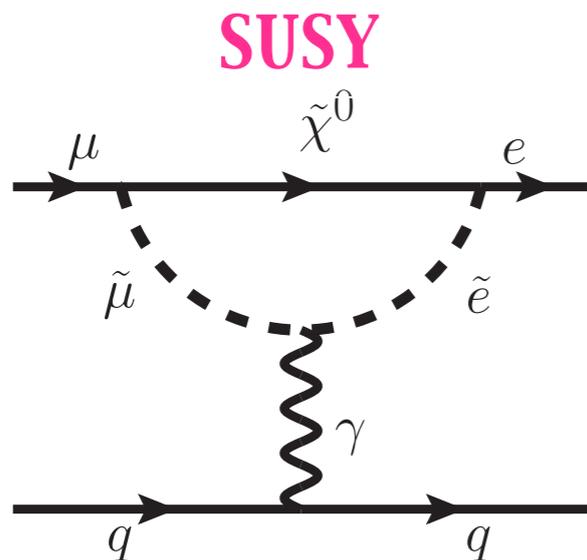
## HADRONIC LIGHT-BY-LIGHT

- ❖ RBC Collaboration [Hayakawa et al., PoS LAT2005 (2006) 353] developed a **promising method using QCD + QED lattice simulations** that is simpler and cleaner than the correlation function of 4 currents, but calculation still in early stages
- ❖ Over the next few years, US lattice-QCD community will **increase both human and computing resources devoted to this high-priority calculation**

# Muon-to-electron conversion



- ◆ Charged-lepton flavor violation highly suppressed in the Standard Model
  - ➔ **Observation of CLFV would be unambiguous evidence of new physics**
- ◆ Many new-physics models allow for CLFV and predict rates close to current limits



- ◆ **Mu2e Experiment @ Fermilab (with Project X)** aims to search for  $\mu N \rightarrow eN$  with a sensitivity four orders of magnitude below the current best limit
- ◆ **MEG@PSI** searching for  $\mu \rightarrow e\gamma$ , while **Mu3e** proposes improved search for  $\mu \rightarrow eee$
- ◆ Combining measured rates of  $\mu \rightarrow e\gamma$  and  $\mu \rightarrow e$  conversion on different target nuclei can distinguish between models and reveal information on underlying theory

# Neutrino physics



- Understanding neutrino-nucleon scattering essential to determine the flux of incoming neutrinos and interpret experimental measurements

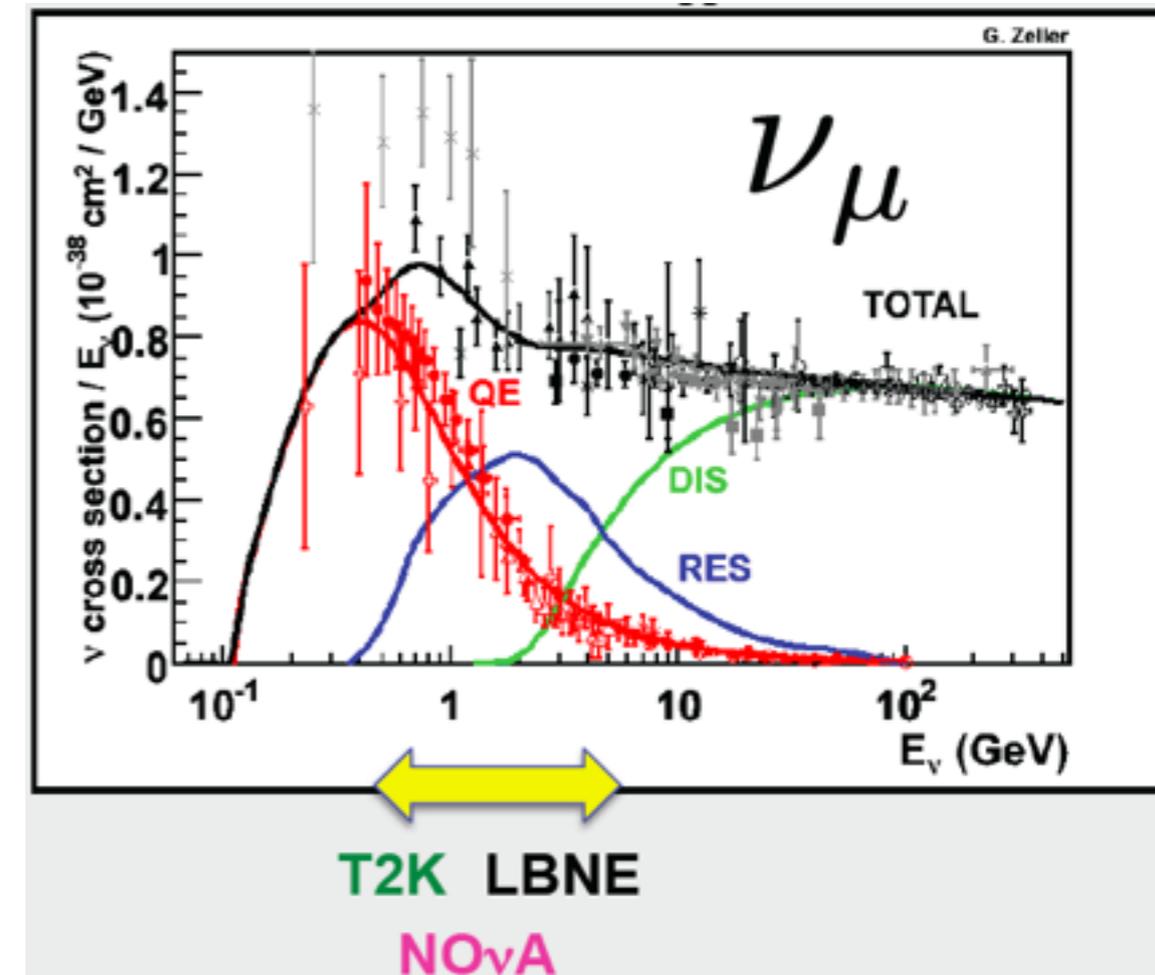
- Accelerator neutrino experiments are in the low-energy regime most complicated by the nuclear environment

- Cross section for quasi-elastic  $\nu_{\mu}n \rightarrow \mu p$  and  $\nu_{\mu}p \rightarrow \mu n$  scattering is parameterized by hadronic form factors that can be computed from first principles with lattice QCD

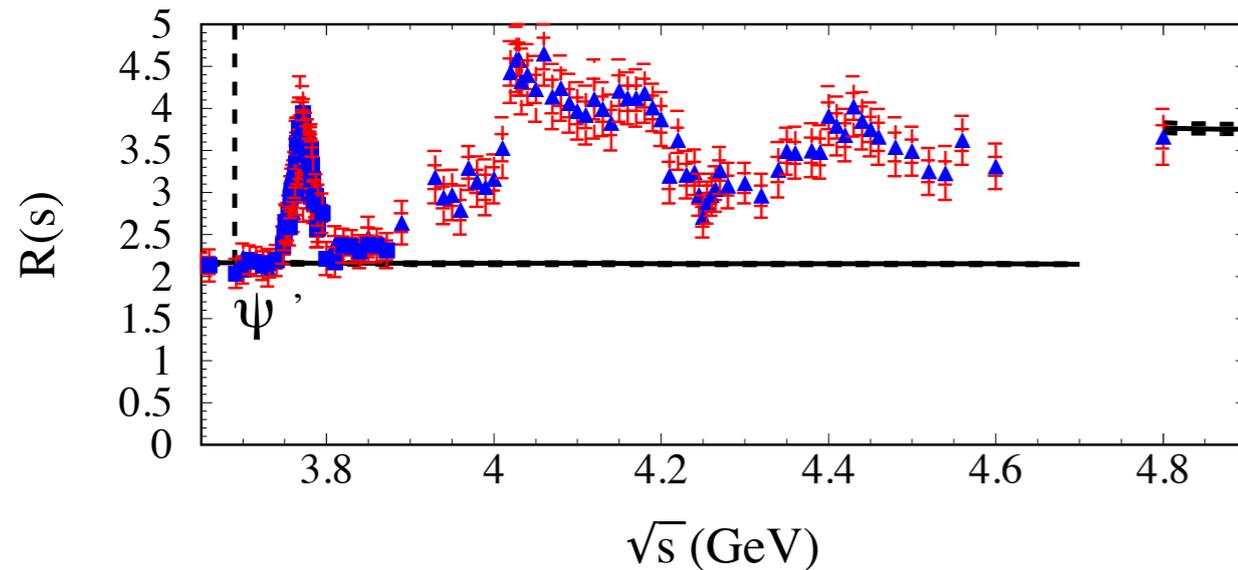
- Observation of proton decay would be unambiguous evidence of new physics

- Lattice-QCD calculations of proton-decay matrix elements  $\langle \pi, K, \eta, \dots | O_{NP} | p \rangle$  essential to interpret limits on (or a measurement of) the proton lifetime as constraints on GUT models

[Formaggio & Zeller, RMP 84, 1307 - 1341, 2012]

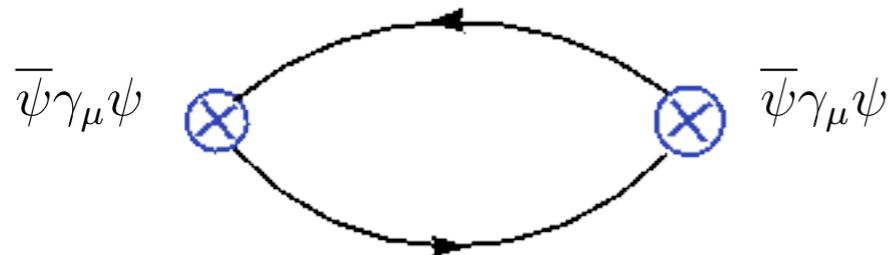


# $e^+e^- \rightarrow m_c$



$$\mathcal{M}_n \equiv \int \frac{ds}{s^{n+1}} R_Q(s)$$

Moments of the heavy quark production cross section in  $e^+e^-$  annihilation can be related to the derivatives of the vacuum polarization at  $q^2=0$ .



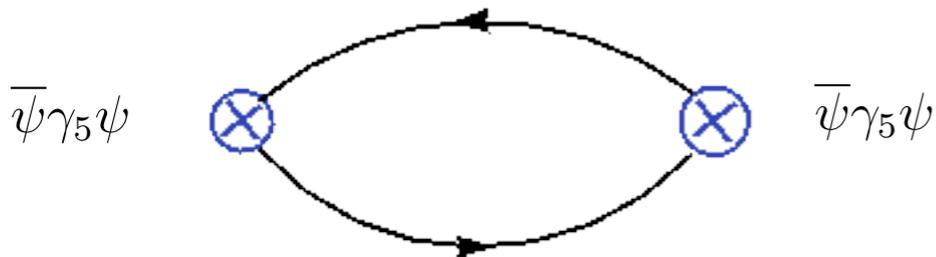
$$\mathcal{M}_n = \frac{12\pi^2}{n!} \left( \frac{d}{dq^2} \right)^n \Pi_Q(q^2) |_{q^2=0}$$

Can be calculated in perturbation theory.  
Known to  $O(\alpha_s^3)$  (Chetyrkin et. al.)

# Lattice QCD

can also compute such correlation functions with high accuracy.

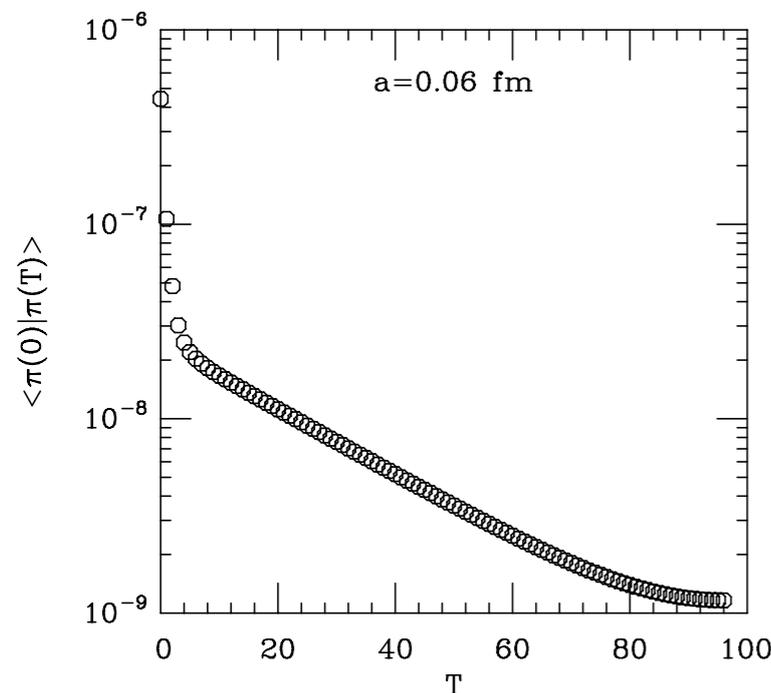
Correlation functions of all currents can be calculated in perturbation theory (and with the lattice). The most precise  $m_c$  can be obtained by choosing the one that is most precise on the lattice: the pseudoscalar correlator.



$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle 0 | j_5(\mathbf{x}, t) j_5(0, 0) | 0 \rangle,$$

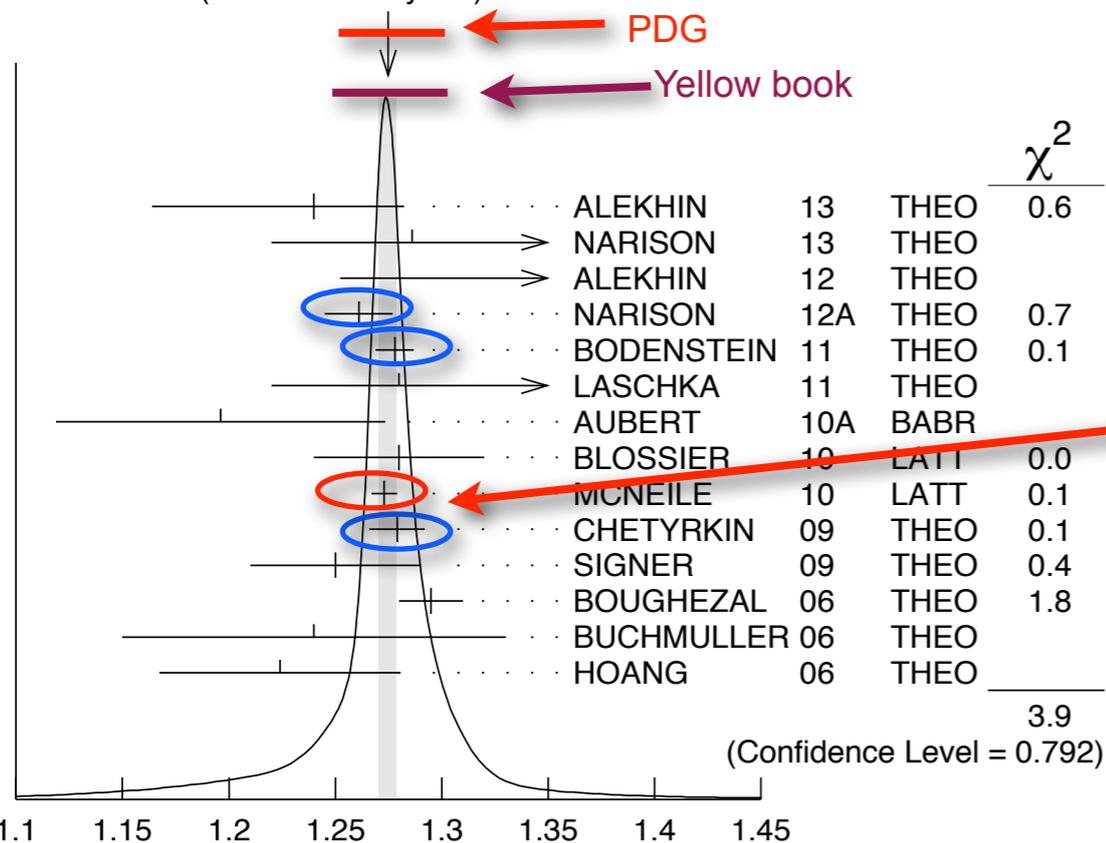
$$G_n \equiv \sum_t (t/a)^n G(t),$$

Perturbation theory to  $\alpha_s^3$  from the Karlsruhe group.



# $m_c$ results

WEIGHTED AVERAGE  
 $1.275 \pm 0.004$  (Error scaled by 1.0)



PDG, Beringer et al., 2013.

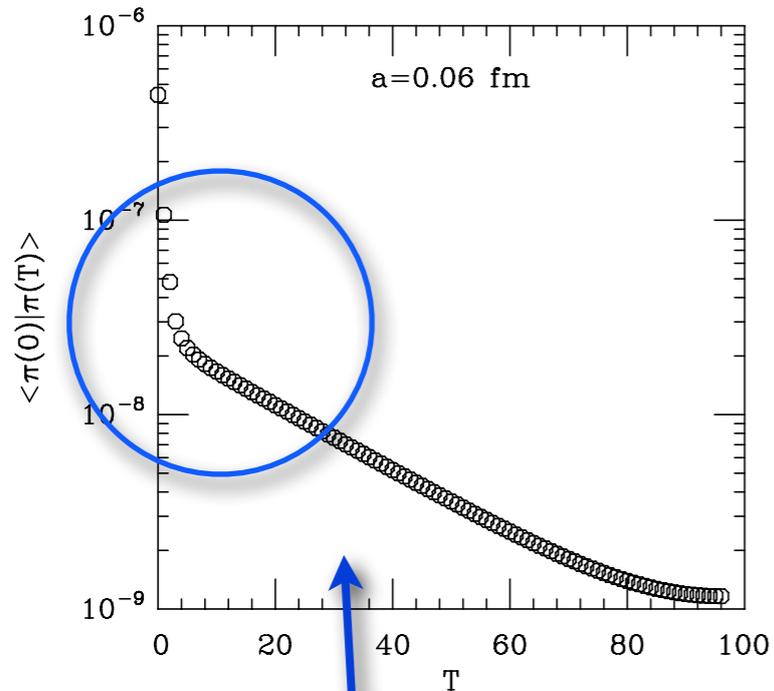
	$m_c(3)$
$a^2$ extrapolation	0.2%
Perturbation theory	0.5
Statistical errors	0.1
$m_h$ extrapolation	0.1
Errors in $r_1$	0.2
Errors in $r_1/a$	0.1
Errors in $m_{\eta_c}, m_{\eta_b}$	0.2
$\alpha_0$ prior	0.1
Glunon condensate	0.0
<b>Total</b>	<b>0.6%</b>

$$m_c(m_c, n_f = 4) = 1.273(6) \text{ GeV.}$$

HPQCD, McNeile et al.

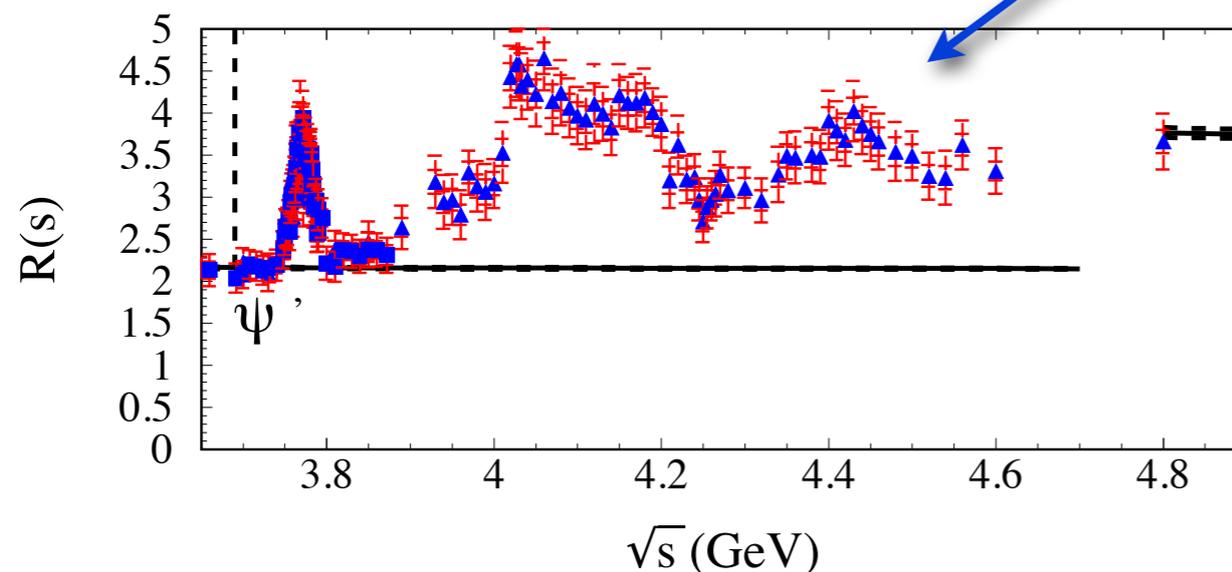
Uncertainty is dominated by the same perturbation theory used in all of the most precise results.

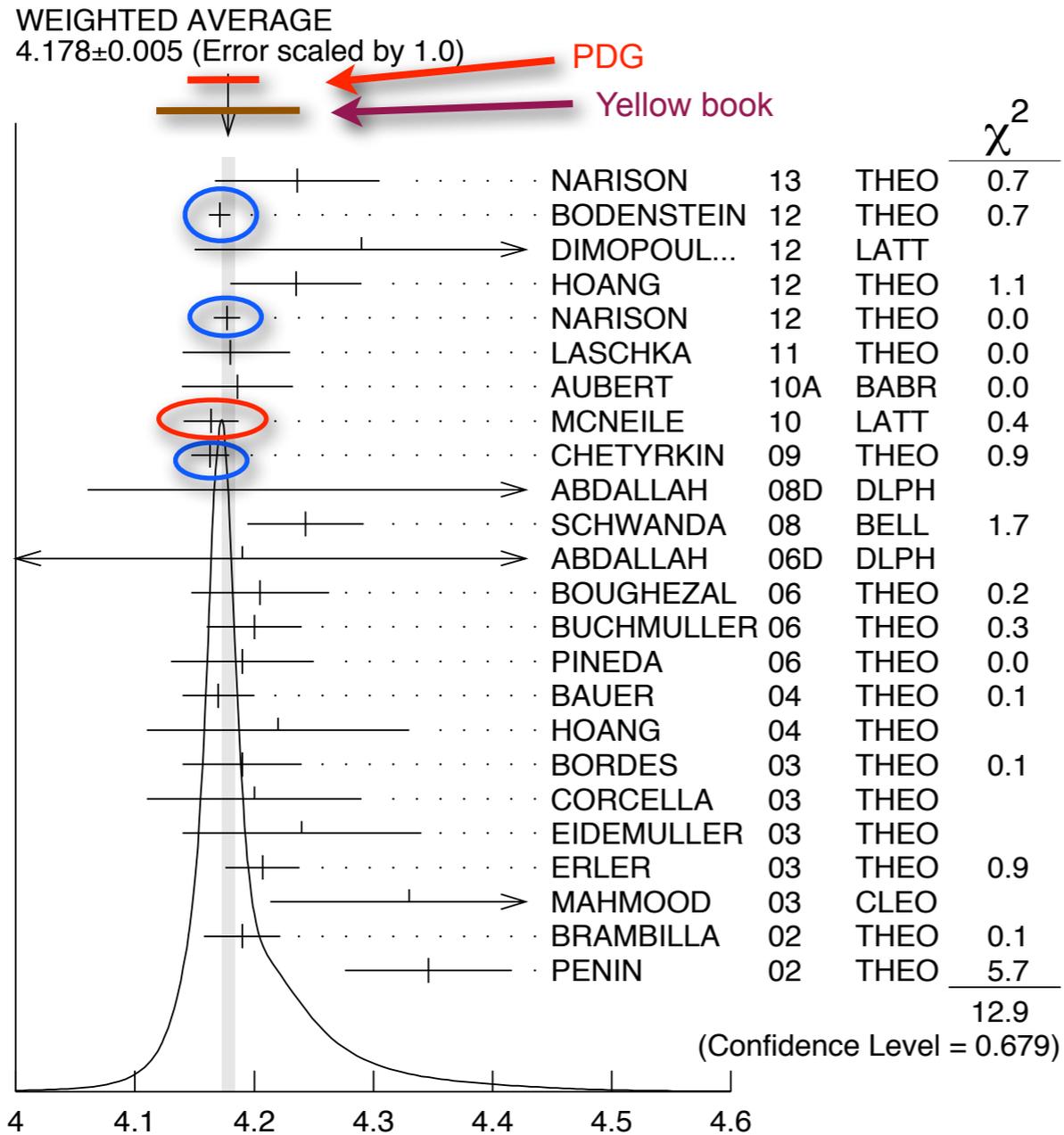
# Why can lattice determinations of $m_c$ from correlation functions be more precise than those from $e^+e^-$ ?



Moments of correlation functions are even easier than what I earlier told you have been considered the easiest quantities for the last ten years. We need the correlation functions at finite  $T$ , and not their asymptotic form at large  $T$ .

Because **this** is cleaner data than **this**.





PDG, Beringer et al., 2013.

The most precise non-lattice determinations of  $m_c$  use  $e^+e^-$  annihilation data and ITEP sum rules. (Karlsruhe group, Chetyrkin et al.)

Recent lattice determination uses the same type of perturbation theory, but lattice QCD to supply the correlation functions rather than experiment.

For  $m_b$ , perturbative errors are tiny. ( $\alpha(m_b)^4 \ll \alpha(m_c)^4$ .)

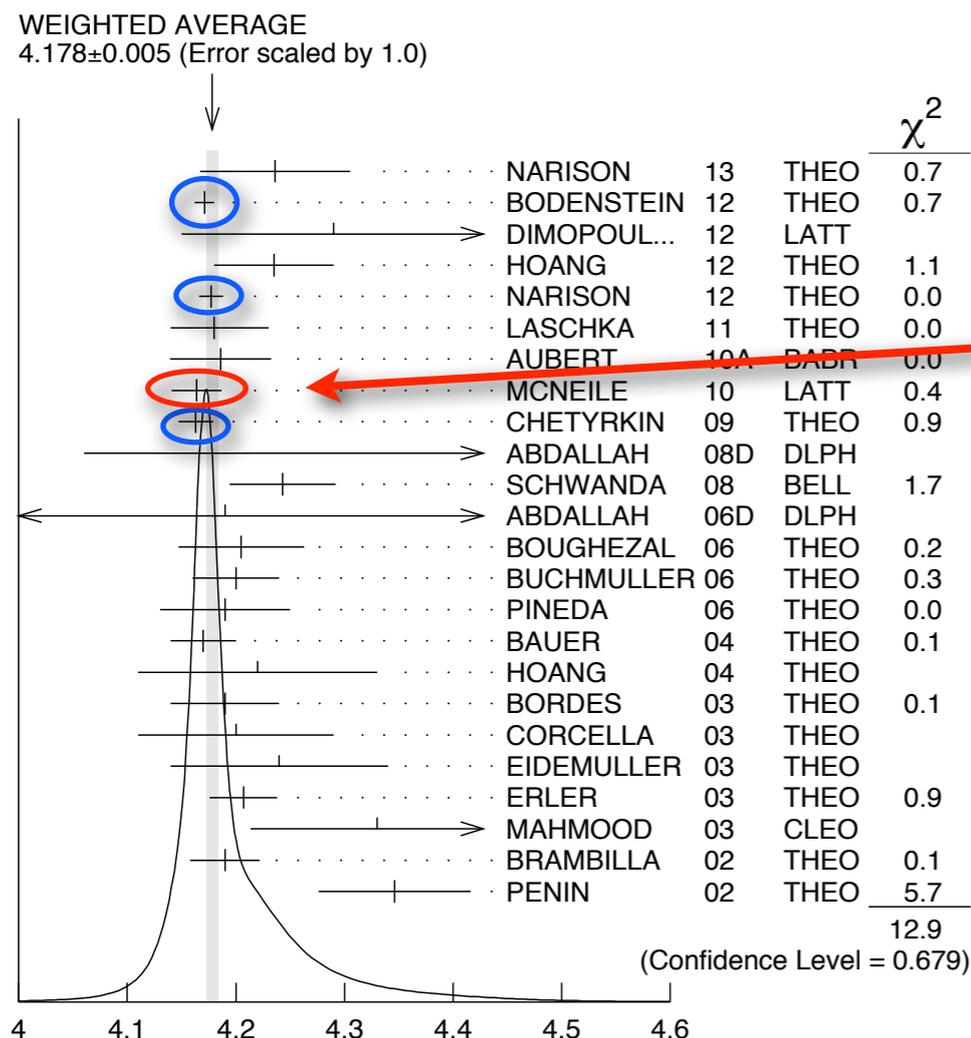
# $m_b$ results

For  $m_b$ , these lattice correlator methods are just barely working at  $a=0.045$  fm. (They treat the  $b$  as light compared with  $1/a$ .)

Need  $a=0.03$  fm to be comfortable.

Discretization errors and statistics dominate current uncertainties. Both can be attacked with brute force computing power.

Needed configurations are projected to be generated in the next few years.



	$m_b(10)$
$a^2$ extrapolation	0.6%
Perturbation theory	0.1
Statistical errors	0.3
$m_h$ extrapolation	0.1
Errors in $r_1$	0.1
Errors in $r_1/a$	0.3
Errors in $m_{\eta_c}, m_{\eta_b}$	0.1
$\alpha_0$ prior	0.1
Gluon condensate	0.0
<b>Total</b>	<b>0.7%</b>

$$m_b(m_b, n_f = 5) = 4.164(23) \text{ GeV}$$

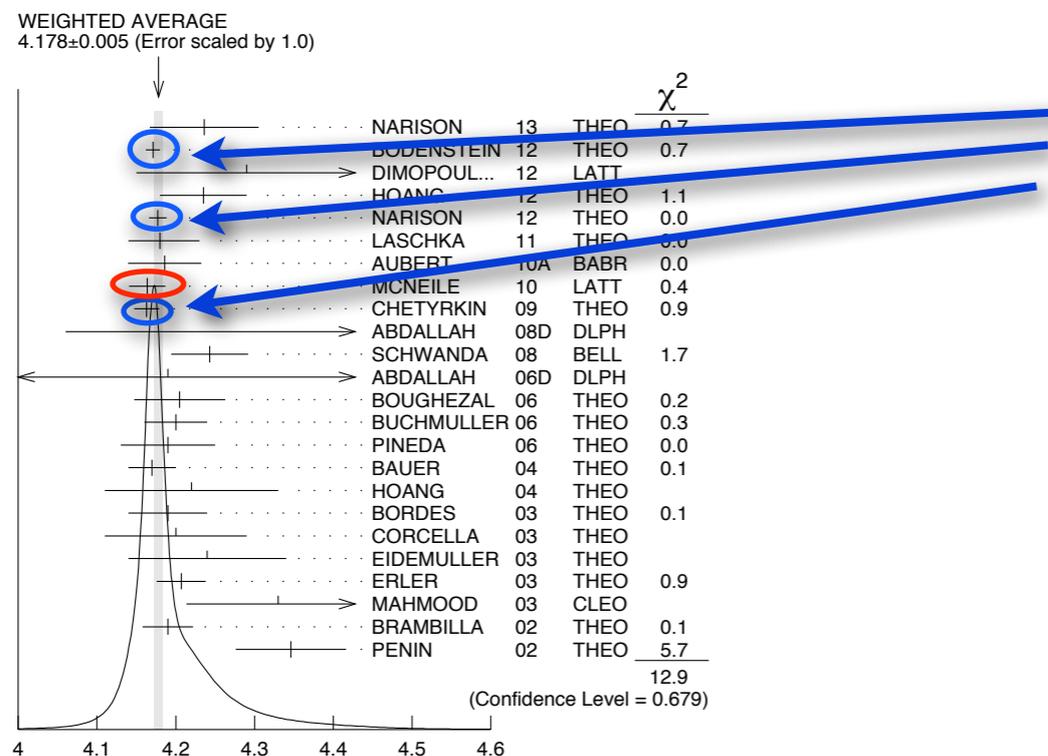
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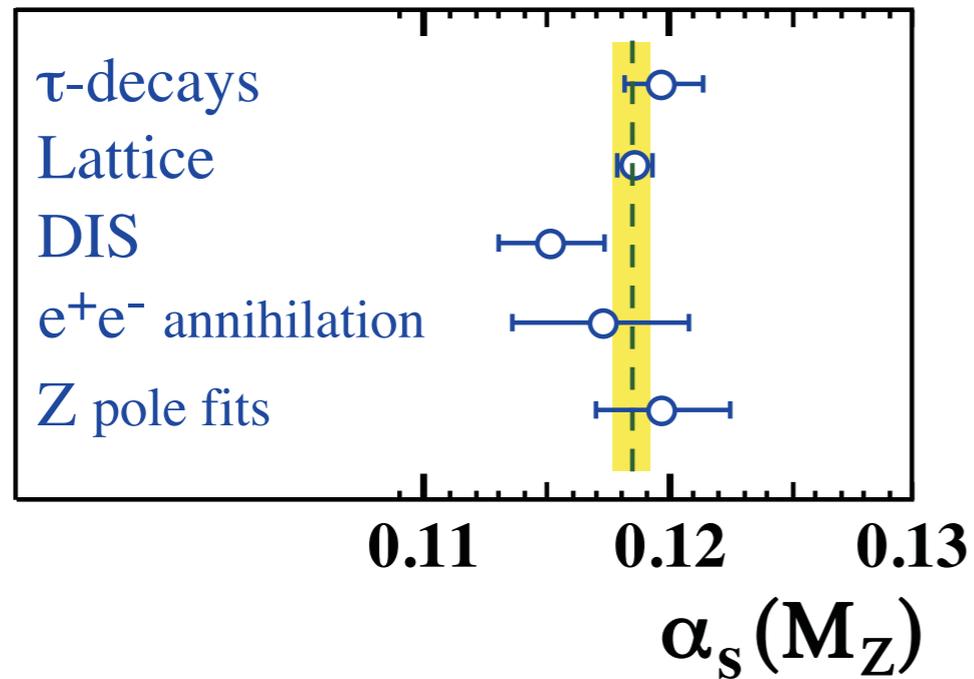


The three most precise determinations of  $m_b$  using moments of  $e^+e^-$  data arrive at different estimates of the precision.

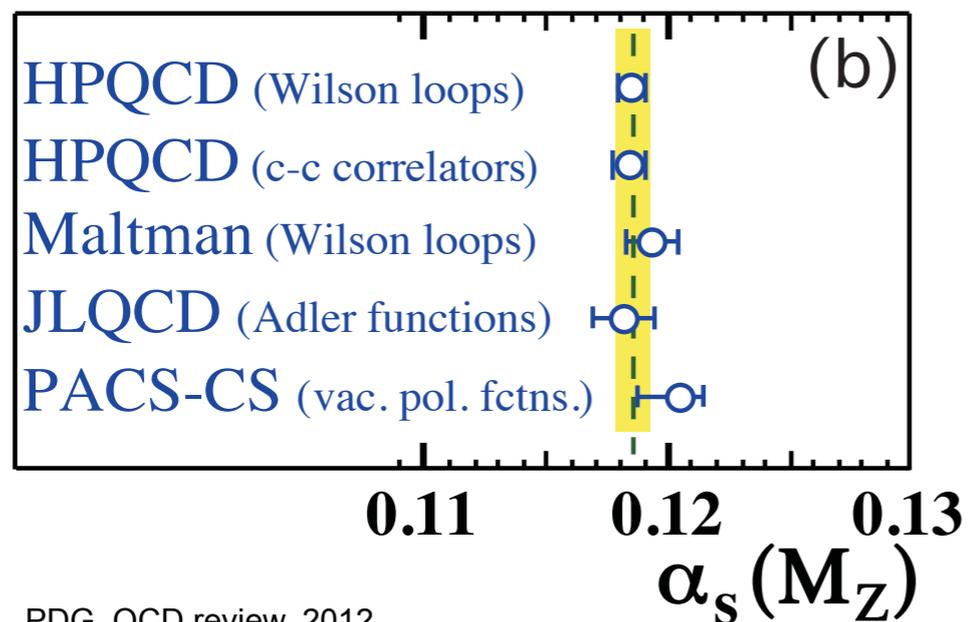
Coming lattice calculations should be able to confirm (or not) the more more precise claims.

Unlike  $m_c$ , where the lattice and  $e^+e^-$  determinations share the same perturbation theory, perturbative uncertainties are negligible and the lattice and  $e^+e^-$  determinations will have totally independent uncertainties.

# $\alpha_s$



There are multiple ways of determining  $\alpha_s$ , both with and without the lattice.

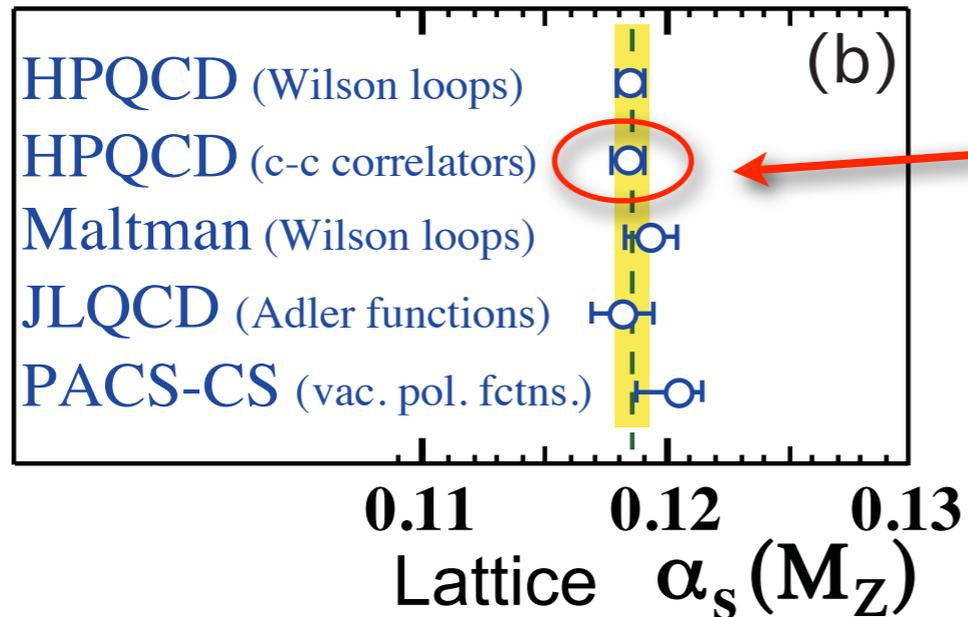


There are several lattice determinations equal to or more precise than all the non-lattice determinations together.

PDG, QCD review, 2012.



# $\alpha_s$ results: correlator method

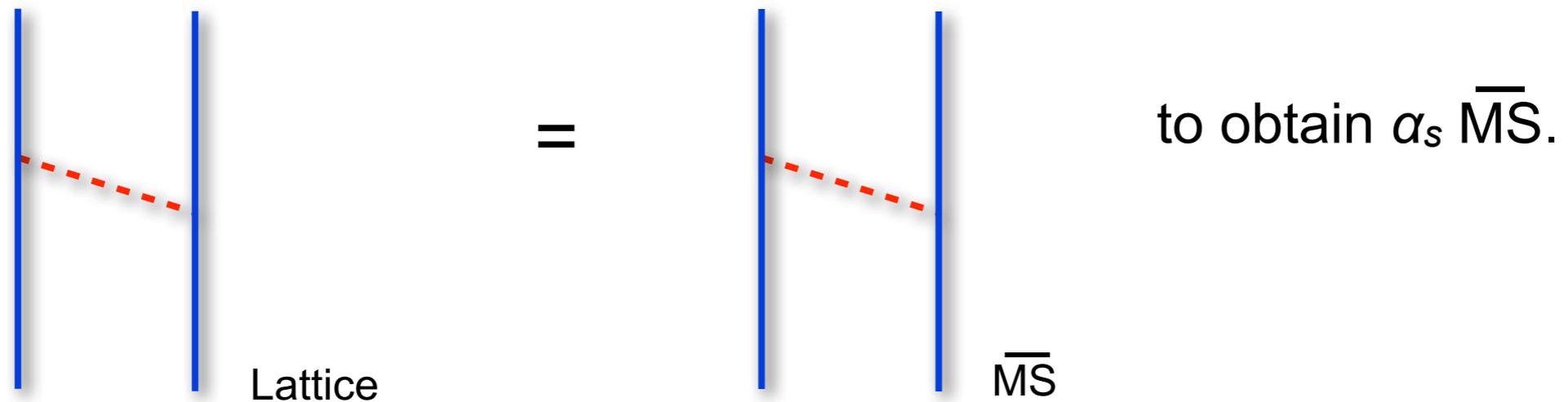


	$\alpha_{\overline{MS}}(M_Z)$
$a^2$ extrapolation	0.2%
Perturbation theory	0.4
Statistical errors	0.2
$m_h$ extrapolation	0.0
Errors in $r_1$	0.1
Errors in $r_1/a$	0.1
Errors in $m_{\eta_c}, m_{\eta_b}$	0.0
$\alpha_0$ prior	0.1
Gluon condensate	0.2
<b>Total</b>	<b>0.6%</b>

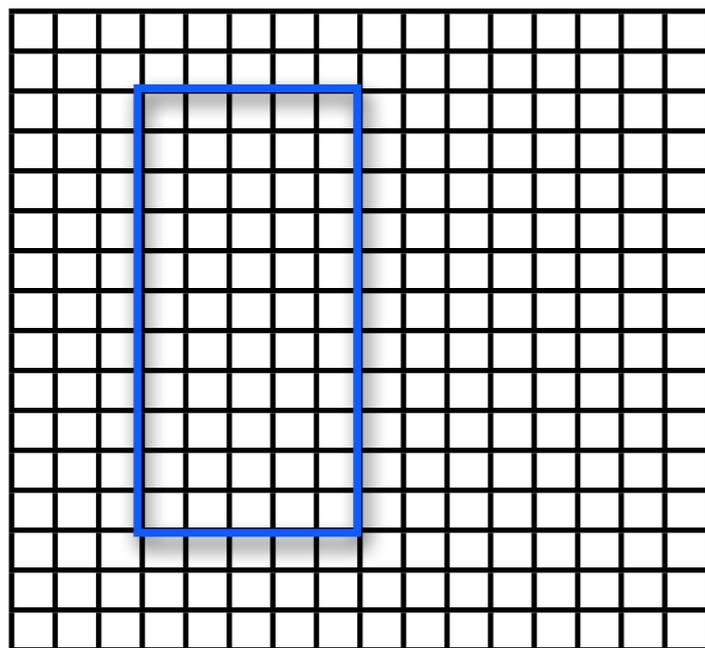
Results are dominated by perturbation theory. May be hard to improve without next term in perturbation theory.

# $\alpha_s$ results: Wilson loops

$\alpha_s$  can be determined with lattice calculations of many other quantities, e.g., the heavy quark potential.



Lattice calculates the heavy quark potential from Wilson loops.



HPQCD has determined  $\alpha_s$  directly from Wilson loops.

Result compatible with their correlator result, similar precision:  $\alpha_s = 0.1184(6)$ , but totally different uncertainties, heavy use of lattice perturbation theory.

# $\alpha_s$ , other lattice results

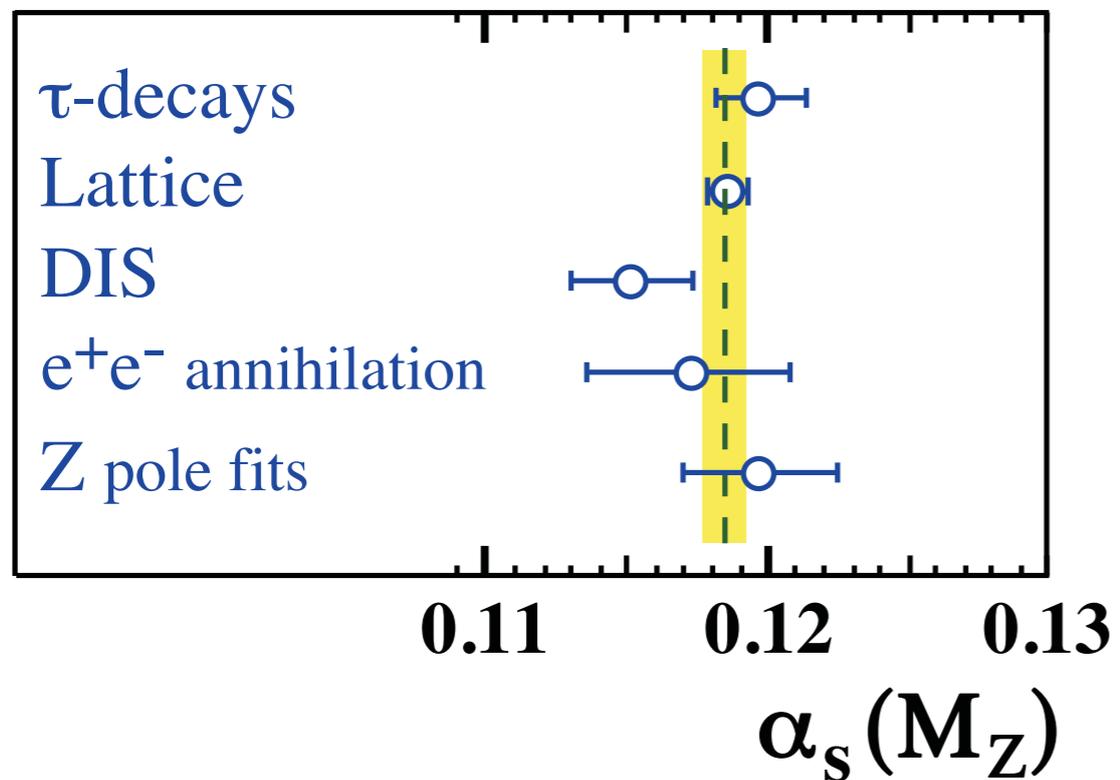
There are numerous good ways of determining  $\alpha_s$  using lattice QCD.

- The Adler function, JLQCD. Phys.Rev. D82 (2010) 074505.
  - $\alpha_s = 0.1181 \pm 0.0003+0.0014-0.0012$
- The Schrödinger functional, PACS-CS. JHEP 0910:053,2009.
  - $\alpha_s = 0.1205(8)(5)(+0/-17)$
- The ghost-gluon vertex, European Twisted Mass Collaboration (ETM). Phys.Rev.Lett. 108 (2012) 262002.
  - $\alpha_s = 0.1200(14)$

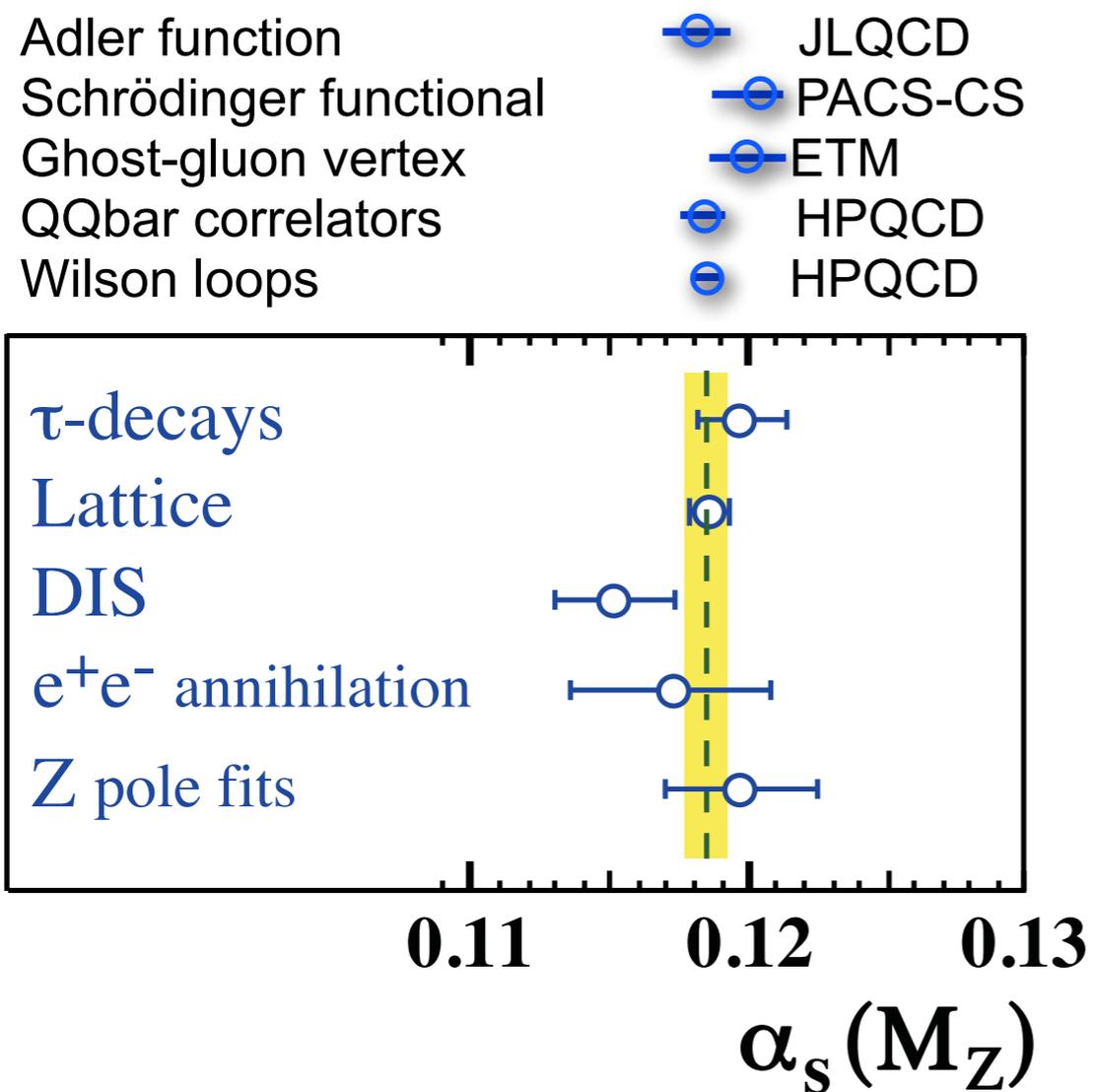


# PDG

2012, combined the lattice numbers in a weighted average. It takes a combined error of the most precise of the inputs.



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The lattice results (2013) are dominated by the two most precise results from HPQCD, but there are several other lattice results from Europe and Japan, all of which agree with each other and each which is more precise than any non-lattice result.