Multiscale coupling in ocean and climate modeling

DOE Office of Science
Applied Mathematics Research program

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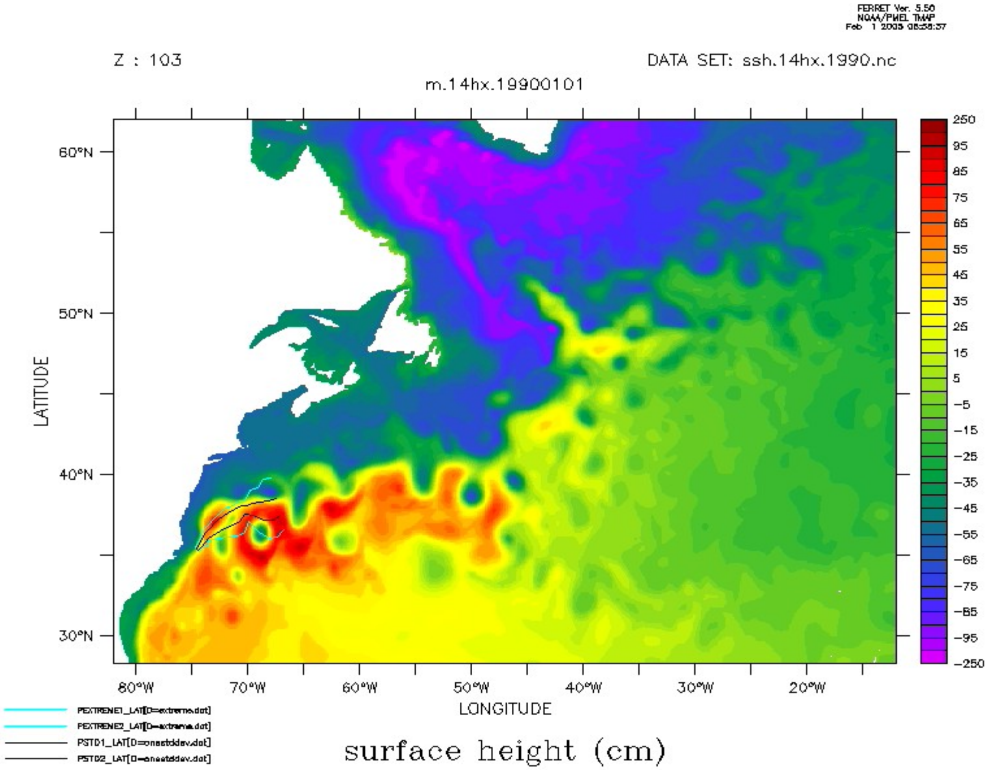
Motivation

- Turbulence parameterization in Geophysical Fluid Dynamics

- Requirements of next generation of ocean/climate models

North-Atlantic simulation 2003

- Parallel Ocean Program, 1/10° resolution, hydrostatic, Rossby deformation radius resolved (L_R~ 10-50km)



- Next step: Non-hydrostatic effects → even smaller eddies

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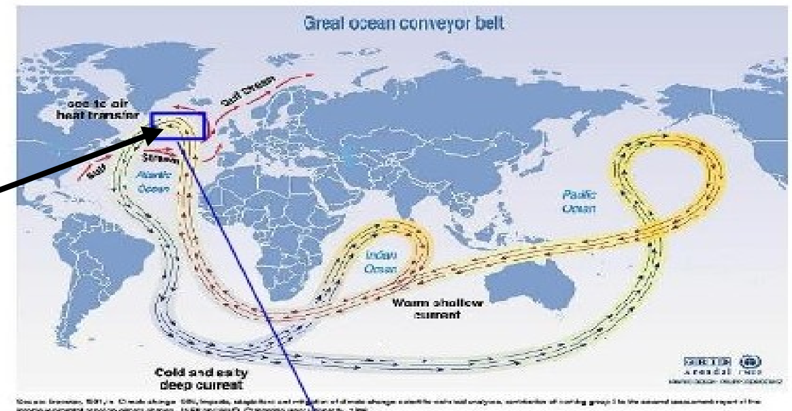
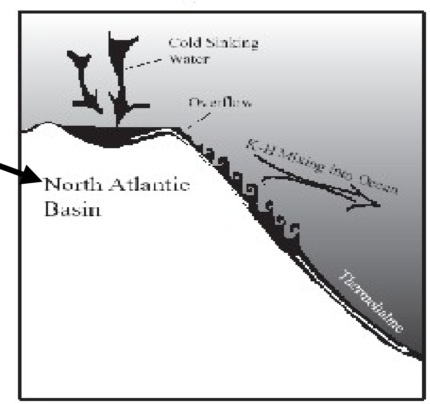
Multiscale coupling in the ocean

Interscale coupling of slow, large scale coherent motions:
  great ocean conveyor belt
and fast, three-dimensional small-scale mixing:
  dense water overflow.





Theoretical understanding, diagnostic tools and parametrizations for the next generation of ocean, atmosphere and climate models

Next generation of models

- Non-hydrostatic small-scale effects are currently ignored, but they have *O(1)* effects over long times.
- Small scale (sub-deformation scale, non-hydrostatic) effects must be explicitly calculated, modeled or parameterized.
- What are the effects of the small scales on the large?

Multiscale coupling in ocean and climate modeling

People

- Susan Kurien, Beth Wingate, Nicole Jeffery (postdoc)

  (Los Alamos National Laboratory)
- Prof. Leslie Smith, Prof. Zhengyu Liu, Jai Sukhatme (postdoc), Mark Remmel (student), Li Wang (student) (University of Wisconsin, Madison)
- Mark Taylor (Sandia National Laboratories)
- Summer students at LANL: Miranda Holmes (Courant Institute of Mathematical Sciences),

  Mike Watson (University of Colorado, Boulder)

Science cornerstones

Connect turbulence and GFD.

- classical turbulence theory does not account for multiple spatial-scale and time-scale dependent parameters (Reynolds (Re), rotation (Ro), stratification (Fr)).
- adapt mathematical tools from turbulence theory to capture the multiscale, multi-parameter nature of GFD.

Scale-linking due to nonhydrostatic effects.

- small-scale vertical mixing and sub-deformation scale effects need to be accounted for in ocean and climate modeling.

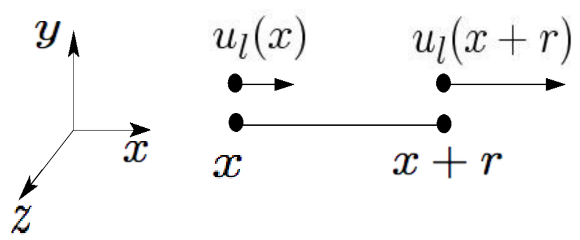
Connecting turbulence and GFD
Key results

- new statistical benchmark for rotating/stratified turbulence (Kurien, Smith & Wingate, J. Fluid Mech., 2006)
- new constraints on energy transfer due to potential enstrophy in strongly rotating/stratified turbulence (Kurien, Wingate, Taylor, to be submitted, 2007)
- ongoing verification of new diagnostics using high-performance DNS code

Non-rotating, non-stratified turbulence

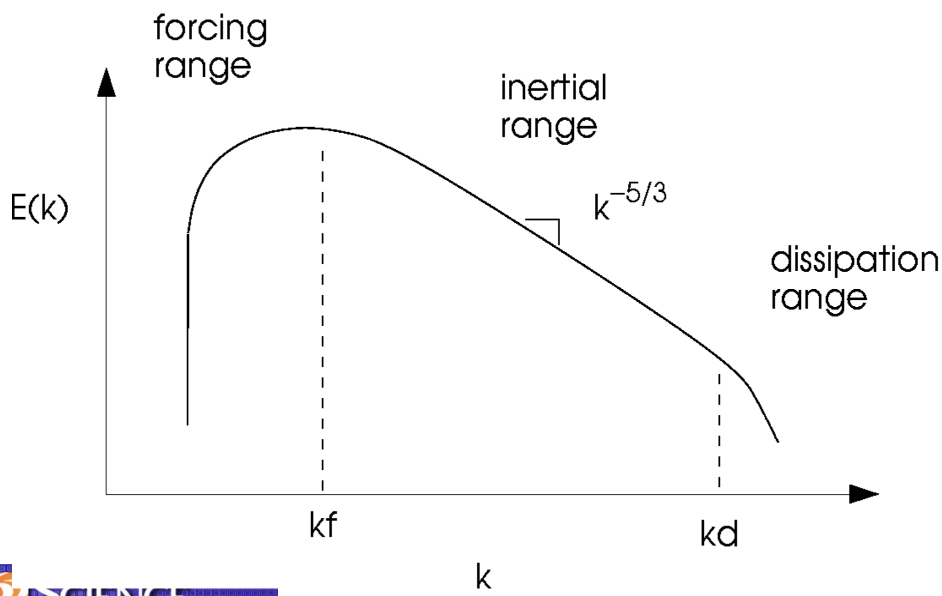
- benchmarks (Kolmogorov 1941)

scale-by-scale energy flux :



$$\Delta u_l = (\boldsymbol{u}(\boldsymbol{x}+\boldsymbol{r}) - \boldsymbol{u}(\boldsymbol{x})) \cdot \hat{\boldsymbol{r}}$$

$$\langle(\Delta u_l(\boldsymbol{r}))^3\rangle = -\frac{4}{5}\varepsilon r \rightarrow E(k) = C\varepsilon^{2/3}k^{-5/3}$$

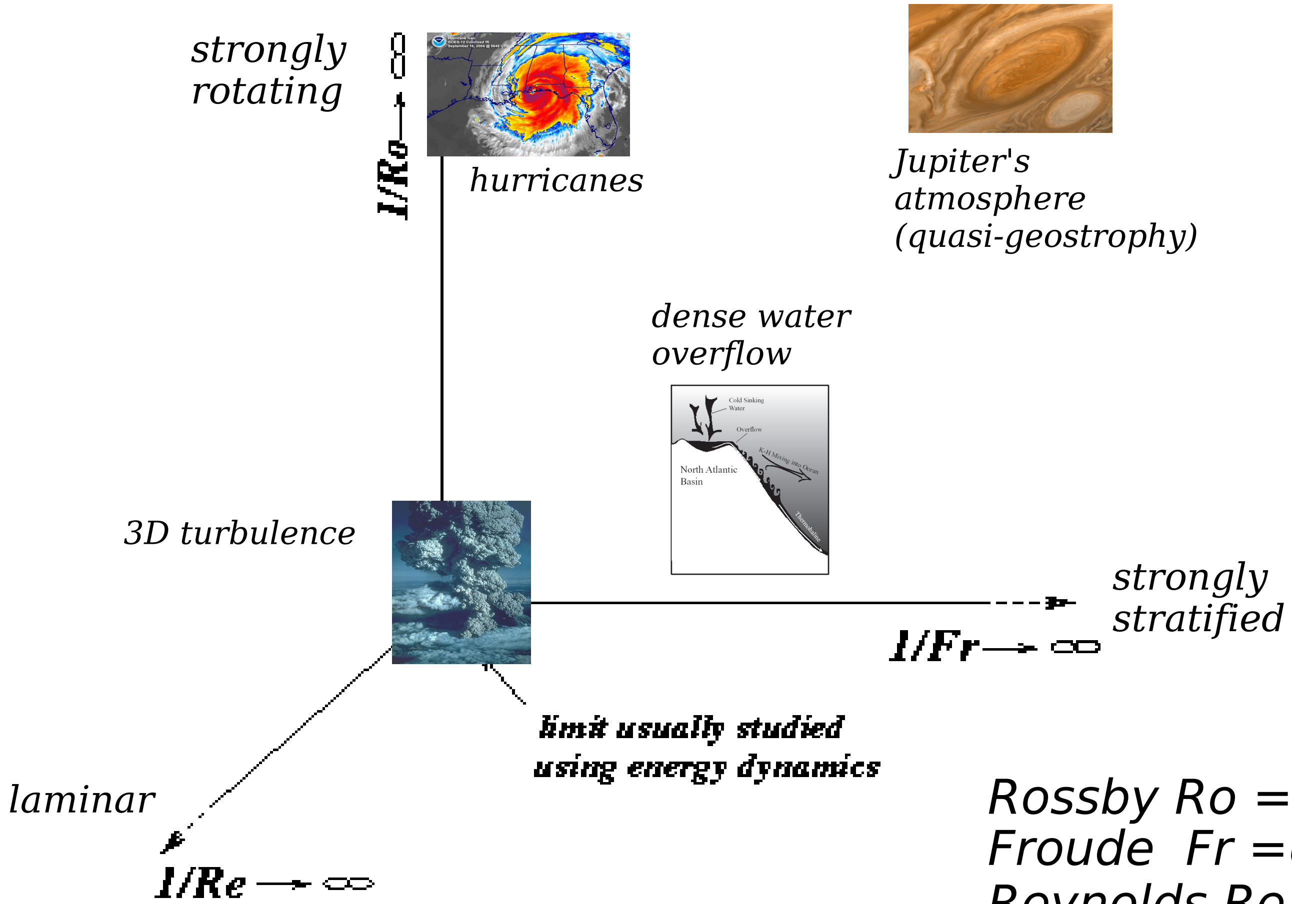


- Reynolds number $Re = UL/\nu$ is the only parameter.
- $Re \rightarrow \infty$ is the only sensible limit.

K41 theory has implications for turbulence modeling

- benchmark for calculations, models and theory.
- physics of the modeled scales can be characterized by K41 statistical parameters
  - eg. Smagorinsky model constant assumes $k^{-5/3}$ scaling of energy spectrum.

Rotating and stratified turbulence has wider parameter space



Rossby $Ro = U/Lf$
Froude $Fr = U/LN$
Reynolds $Re = UL/\nu$

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Beyond energy -- dependence on *Ro* and *Fr*

- potential vorticity locally conserved

$$q = \boldsymbol{\omega}_a \cdot \nabla \rho$$

$$\boldsymbol{\omega}_a = \boldsymbol{\omega} + 2\boldsymbol{\Omega}$$

$$\rho = \rho_0 + bz + \tilde{\rho}$$

non-dimensional form, *Ro* and *Fr* dependence

$$q = \boldsymbol{\omega} \cdot \nabla \tilde{\rho} + Ro^{-1} \frac{\partial \tilde{\rho}}{\partial z} - Fr^{-1} \omega_3$$

- potential enstrophy conserved

$$Q = q^2$$

Potential enstrophy and energy cascades: Quasi-geostrophy

- *Approximation* for large-scale rotating and stratified flow, assumes small scales are dynamically unimportant.
- Charney (1971) : Potential enstrophy conservation suppresses forward cascade of energy, and scaling of energy spectrum in the high wave numbers :

$$E(k) = \frac{1}{2} \sum_{|k'|=|k|} |\tilde{\boldsymbol{u}}(\boldsymbol{k}')|^2 \sim k^{-3}$$

$$P(k) = \frac{1}{2} \sum_{|k'|=|k|} |\tilde{\theta}(\boldsymbol{k}')|^2 \sim k^{-3}$$

Potential enstrophy and energy away from quasi-geostrophy

- Our new results begin to include small-scale effects in rotating and stratified flows
  - law for flux of potential enstrophy,

$$\langle qq'(u_l' - u_l)\rangle = -\frac{2}{3}\varepsilon_Q r$$

(Kurien, Smith & Wingate (2006))

  - scaling laws for potential energy and horizontal kinetic energy spectra,

(Kurien,Wingate & Taylor (2007))

$$Case\ \frac{k_h}{k_z} \ll 1 \quad P(k_h, k_z) \sim \varepsilon_Q^{2/5} k_z^{-3}$$

$$Case\ \frac{k_z}{k_h} \ll 1 \quad E_h(k_h, k_z) \sim \varepsilon_Q^{2/5} k_h^{-3}$$

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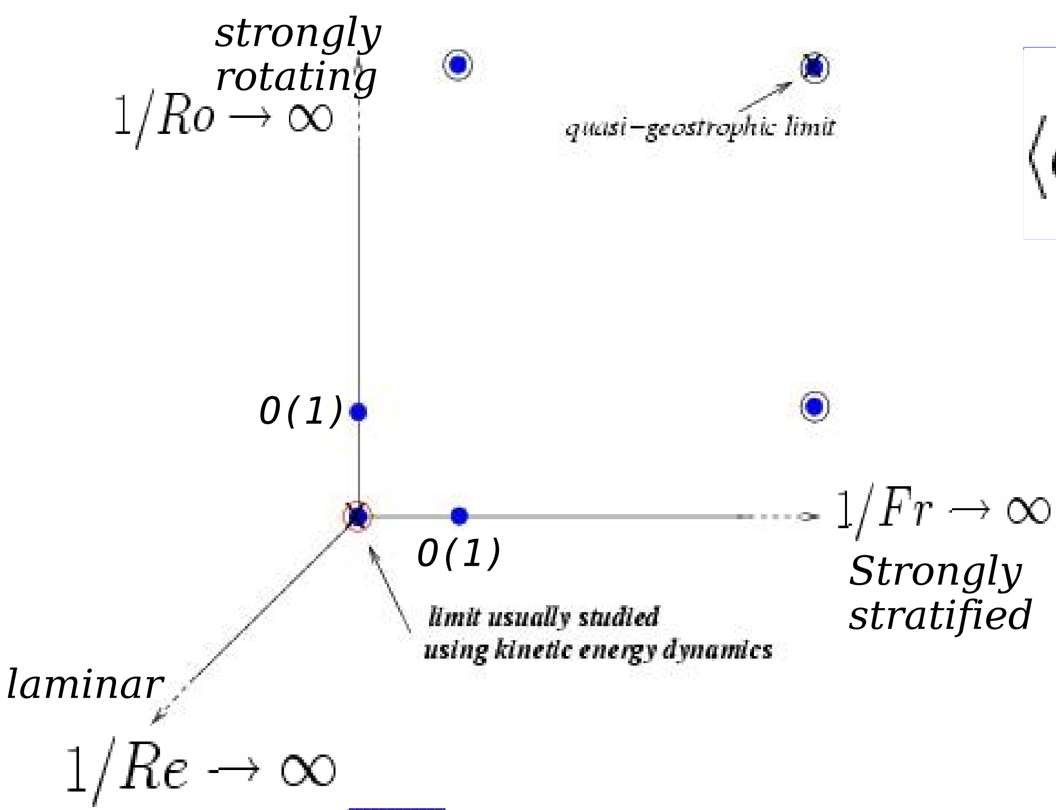
Statistical law for potential enstrophy flux ε_Q

- : six different limits in Ro and Fr

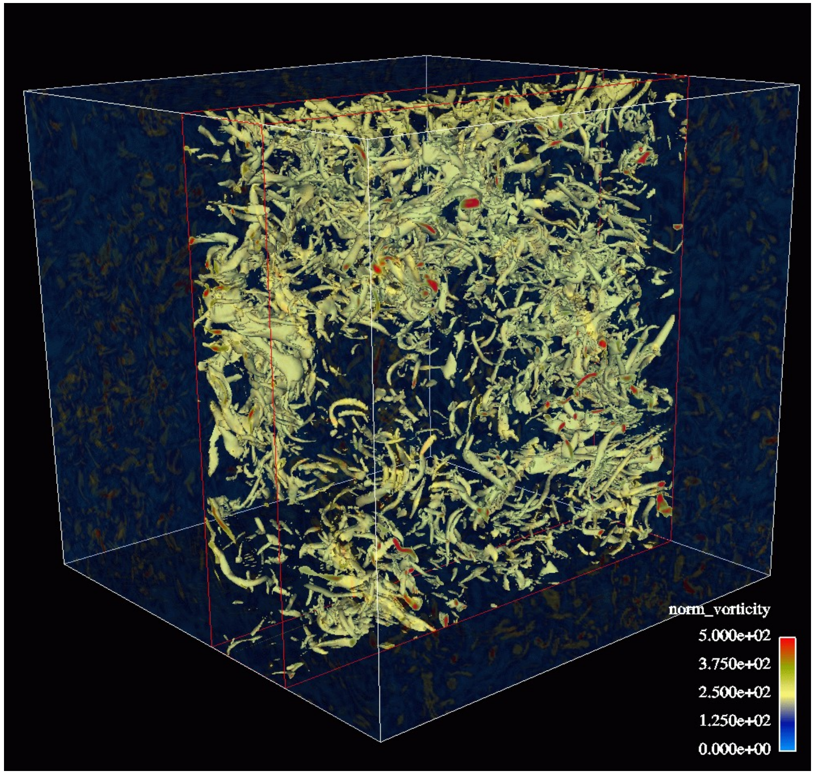
o : new `2/3-law' law for potential enstrophy flux



$$\langle q q' (u_l' - u_l) \rangle = -\frac{2}{3} \varepsilon_Q r$$

analogous to K41 `4/5-law' for kinetic energy flux

LANL high-performance DNS code



256^3 section of 2048^3 simulation of decaying turbulence on ASC-Q
(Mark Taylor, 2003)

More on data and diagnostic techniques:
S. Kurien and M.A. Taylor, Los Alamos Science (2005)

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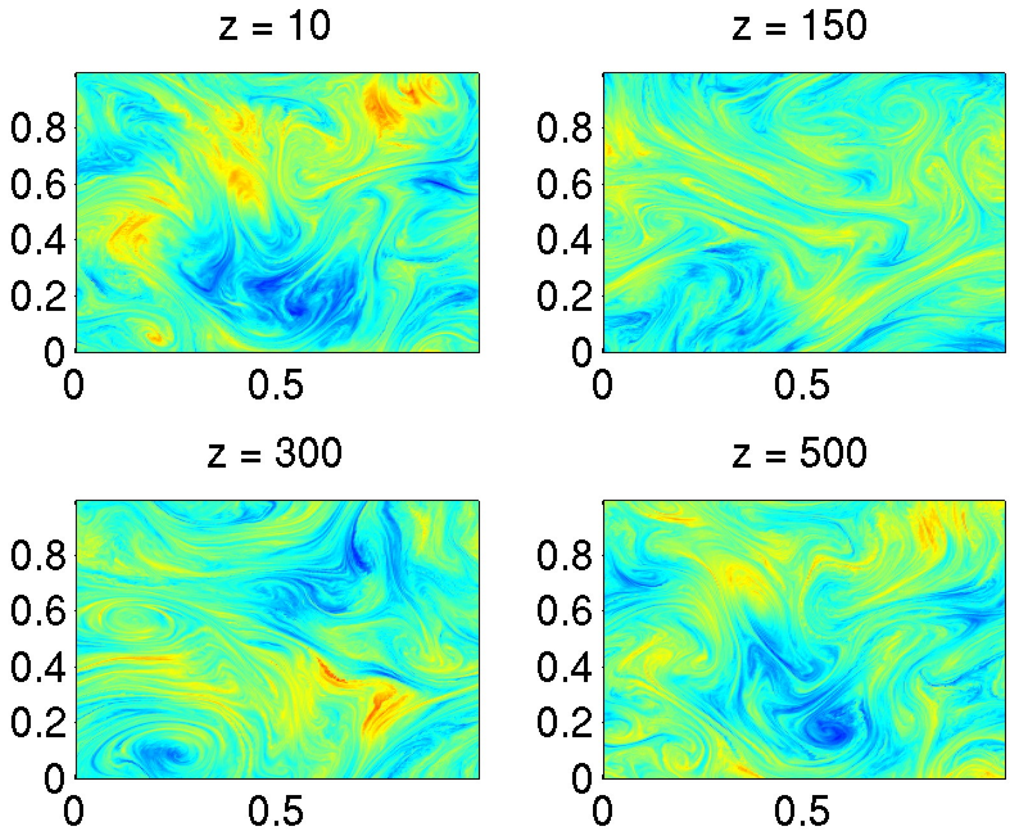
Numerical simulations of rotating and stratified turbulence

- periodic box, uniform grid, 512 gridpoints per side
- rotating and stratified in *z*-direction
- unit aspect ratio
- Boussinesq equations : $\rho << \rho_0$
- stochastically forced at wavenumber $k_f = 4$
- tunable *Ro, Fr, Pr, Re* and aspect ratio
- results from (Ro, Fr) ~ 0.001 (very rapidly rotating and stably stratified)

Scalar field at various heights

- vertically coherent structures emerge



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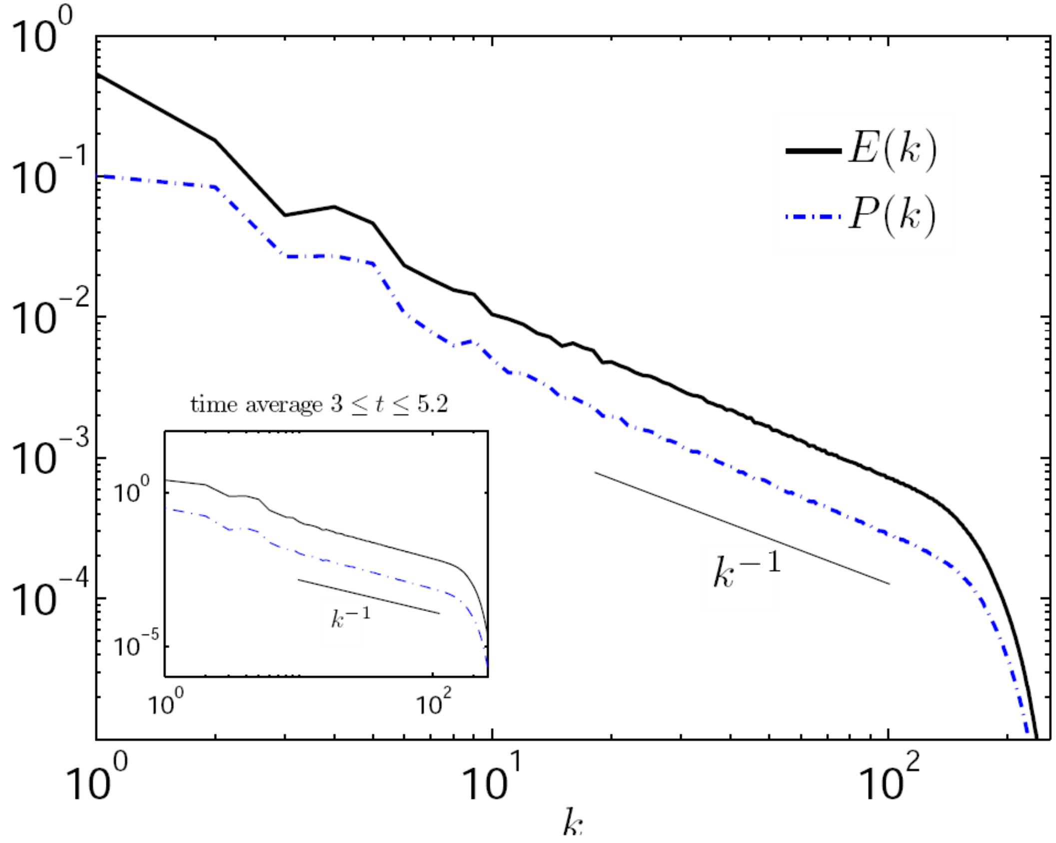
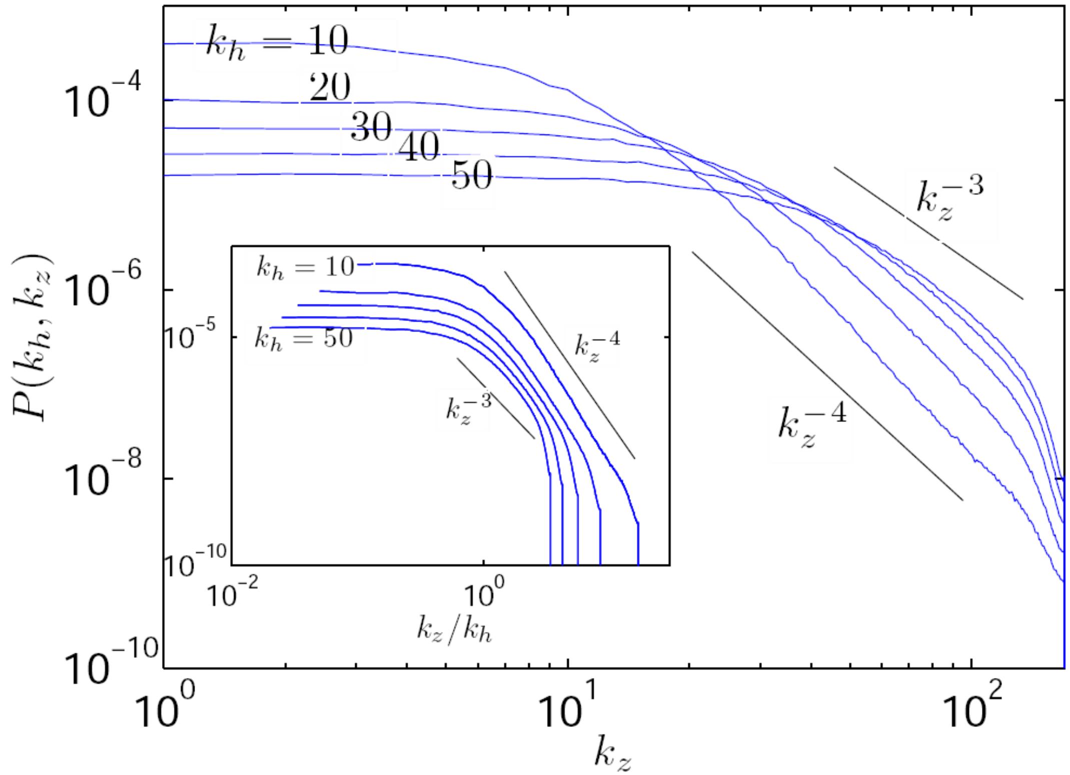
- Charney scaling not observed, flow is not QG since small-scale waves are retained.



- potential enstrophy suppresses potential energy in the nearly vertical modes

$$Case\ \frac{k_h}{k_z} \ll 1 \qquad P(k_h, k_z) \sim \varepsilon_Q^{2/5} k_z^{-3}$$



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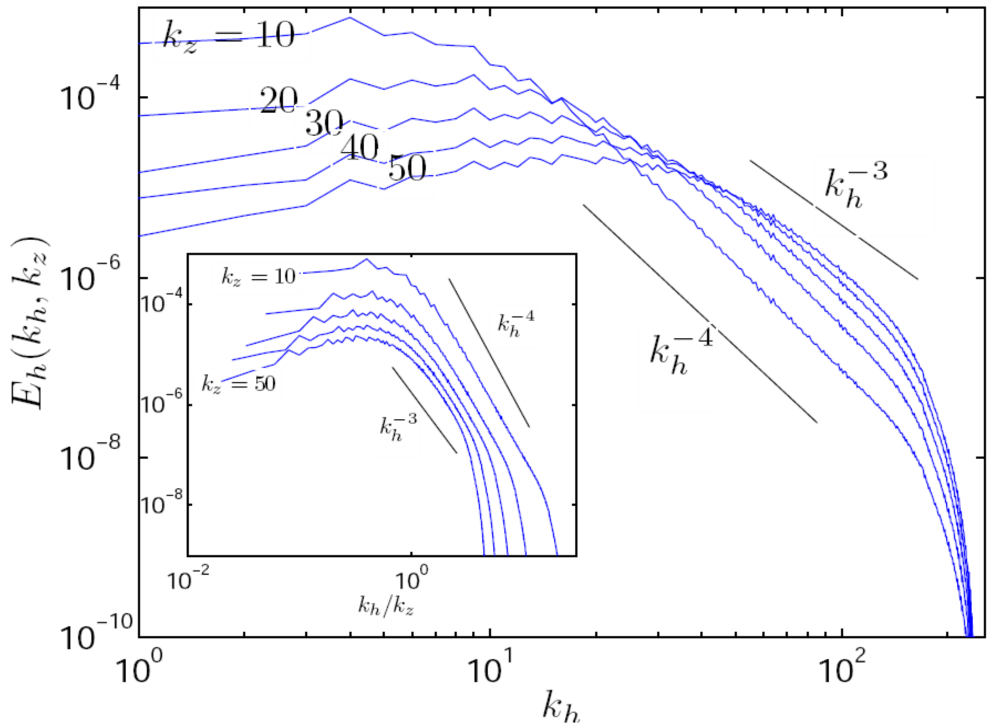
- potential enstrophy suppresses horizontal kinetic energy in the nearly horizontal modes

$$Case\ \frac{k_z}{k_h} \ll 1 \qquad E_h(k_h, k_z) \sim \varepsilon_Q^{2/5} k_h^{-3}$$



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Back to original motivation

- Turbulence parameterization in GFD
  - benchmark law for potential enstrophy flux:

    new "2/3-law" in a wide parameter space (Rossby, Froude values).
  - characterization of the small-scales, *away from classical QG:* derived predictable scaling exponents for energy due to constraining effect of potential enstrophy.
- New results anticipate the next generation of models